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SOME COMMON BASIC PROGRAMS

3RD EDITION

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SOME COMMON BASIC PROGRAMS, 3RD EDITION

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Table of Contents

Program	Page
Future Value of An Investment	1
Future Value of Regular Deposits (Annuity)	3
Regular Deposits	5
Regular Withdrawals from an Investment	7
Initial Investment	9
Minimum Investment for Withdrawals	11
Nominal Interest Rate on Investments	13
Effective Interest Rate on Investments	15
Earned Interest Table	17
Depreciation Rate	22
Depreciation Amount	23
Salvage Value	25
Discount Commercial Paper	27
Principal on a Loan	29
Regular Payment on a Loan	31
Last Payment on a Loan	33
Remaining Balance on a Loan	36
Term of a Loan	38
Annual Interest Rate on a Loan	41
Mortgage Amortization Table	44
Greatest Common Denominator	49
Prime Factors of Integers	51
Area of a Polygon	52
Parts of a Triangle	54
Analysis of Two Vectors	58
Operations on Two Vectors	60
Angle Conversion: Radians to Degrees	61
Angle Conversion: Degrees to Radians	63
Coordinate Conversion	65
Coordinate Plot	67
Plot of Polar Equation	72
Plot of Functions	77
Linear Interpolation	81
Curvilinear Interpolation	83
Integration: Simpson's Rule	86
Integration: Trapezoidal Rule	88
Integration: Gaussian Quadrature	90
Derivative	92

Table of Contents (Continued)

Program	Page
Roots of Quadratic Equations	93
Real Roots of Polynomials: Newton	95
Roots of Polynomials: Half-interval Search	97
Trig Polynomial	99
Simultaneous Equations	101
Linear Programming	103
Matrix Addition, Subtraction, Scalar Multiplication	108
Matrix Multiplication	111
Matrix Inversion	114
Permutations and Combinations	116
Mann-Whitney *U* Test	118
Mean, Variance, Standard Deviation	121
Geometric Mean and Deviation	124
Binomial Distribution	125
Poisson Distribution	127
Normal Distribution	128
Chi-square Distribution	130
Chi-square Test	133
Student's *t*-distribution	135
Student's *t*-distribution Test	137
F-distribution	140
Linear Correlation Coefficient	143
Linear Regression	145
Multiple Linear Regression	147
N th Order Regression	151
Geometric Regression	154
Exponential Regression	156
System Reliability	158
Average Growth Rate, Future Projections	160
Federal Withholding Taxes	162
Tax Depreciation Schedule	165
Check Writer	169
Recipe Cost	173
Survey Check (Map Check)	177
Day of the Week	184
Days between Two Dates	186
Anglo to Metric	189
Alphabetize	192

Introduction

This book describes a number of programs, written in the BASIC[1] programming language. These programs perform a variety of common, practical tasks. The programs are written in a restricted subset of standard BASIC that is compatible with many versions of BASIC now available to microcomputer users.

You can use this book whether or not you know how to write programs in BASIC.

We do not teach you how to program in BASIC; there are probably hundreds of books trying to do that. But we do describe progams carefully and include user examples with the program listings. So if you are not familiar with BASIC, simply copy the program listings into your computer; then run the programs as illustrated in the examples.

Remarks are included in the listings to help BASIC programmers understand how each program works. They will also assist you in identifying parts of programs that you may be able to use in other programs you write. Remark statements precede the line(s) on which they comment. *REM statements should be omitted when you enter programs, since they are ignored by the computer and simply use up memory.*

Options are also included with some programs. An option is an alteration which changes the input or output format of the original program. Options may suggest ways in which you can further alter the programs. We have included a brief description, example, sample run and partial listing of each option. The partial listing includes those program statements which are changed when going from the original program to the optional program. Lines which must be altered, added or deleted are shaded in both listings.

All programs can be run using a Teletype[2] or similar input/output device with a line width as short as 72 characters. If the line width on your output device is less than 72 characters, you may want to alter the print statements within programs that print longer lines.

Certain programs will require additional programming if you use a CRT display or separate printing device for output. If using a CRT, you will probably want to put a pause in some programs after displaying one screenful of data; otherwise, the data will be displayed faster than you can read it. If using a separate printing device for your output, you may need to add print device select statements to the programs.

All programs in this book have been tested, run and listed on a Wang 2200 computer system. They have also been tested and run on a Commodore PET[3]. Some of this testing was with programs modified for better efficiency on the PET.

BASIC Compatibility

Readers of the first and second editions of this book have helped locate typographical and programming errors and have informed us of some compatibility problems for users with different BASICs. For this third edition we have done some reprogramming to eliminate errors and help solve the compatibility problems. Even so, you should be aware of some general compatibility problems which may occur.

1) Some programs may branch out of a FOR/NEXT loop before its iteration is complete. If branching out in the middle of a FOR/NEXT loop causes an error in your BASIC, you can change the FOR/NEXT loop at that point to branch to the NEXT statement, avoiding any program statements which would alter other variable values as the loop completes itself.

2) When a FOR/NEXT loop is completed, the index variable should remain set to its highest value when the progam resumes after the loop. For example, in the following program the value of I at line 30 should be 10.

```
10 FOR I = 1 TO 10
20 NEXT I
30 PRINT I
```

[1]BASIC is a registered trademark of the Trustees of Dartmouth College.
[2]Teletype is a registered trademark of the Teletype Corporation.
[3]PET is a registered trademark of Commodore, Inc.

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2) When a FOR/NEXT loop is completed, the index variable should remain set to its highest value when the progam resumes after the loop. For example, in the following program the value of I at line 30 should be 10.

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10 FOR I = 1 TO 10
20 NEXT I
30 PRINT I
```

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[3]PET is a registered trademark of Commodore, Inc.

Your BASIC may say I = 11 at line 30. To fix this, add a new line after the NEXT statement to decrement the index variable by one. In the above program, add the following:

```
25 I = I - 1
```

If you have a BASIC which does *not* have a FOR/NEXT index variable keep its value outside the loop, you will have to set the variable equal to the end value of the loop once outside the loop (you need to do this only if that index variable value is used later in the program).

3) One variation of the RESTORE command is RESTORE *n*, where *n* refers to the n^{th} item on the data list, and this n^{th} item is to be read in the next READ statement. If your BASIC does not accept RESTORE *n*, change that part of the program to RESTORE, then loop to read data to the n^{th} item.

4) If your BASIC does not set all variables to zero for each RUN (unless another value is specified in a statement), then you will have to write statements to initialize each variable to zero at the beginning of each program.

5) Some programs use DEF FNR (). If not implemented, substitute this expression (with appropriate variable) for each callout of FNR.

6) Some programs use the TAB function with the PRINT statement. You can usually replace it with a string of blanks of the appropriate length. For instance, PRINT TAB (5); A1 could be PRINT " "; A1.

Program Errors

If you encounter an error or program difficulty which you believe is not your fault, we would like to hear about it. Please write the authors in care of the publishers, and include the followng information:

1) description of the error
2) data entered which caused the error
3) source listing of your program, from your computer (if possible)
4) any corrections you have

We appreciate your help in creating a book of tested BASIC programs that *anyone* can use.

Future Value of an Investment

This program calculates a future value of an investment when interest is a factor. You must provide the amount of the initial investment, the nominal interest rate, the number of compounding periods per year and the number of years of investment.

Assuming there are no additional deposits and no withdrawals, the future value is based on the following formula:

$$T = P(1 + i/N)^{N \cdot Y}$$

where: T = total value after Y years (future value)
P = initial investment
i = nominal interest rate
N = number of compounding period per year
Y = number of years

Examples:

Carl makes an investment of $6800.00 at 9.5%. If interest is compounded quarterly, what will be the value of Carl's investment in 10 years?

Mr. Smith purchases a piece of property for $16,050.00. The value of property is rising at an average of 7% per year. What may Mr. Smith expect his property to be worth in 5½ years?

```
:RUN
FUTURE VALUE OF AN INVESTMENT

INITIAL INVESTMENT? 6800
NOMINAL INTEREST RATE? 9.5
NUMBER OF COMPOUNDING PERIODS PER YEAR? 4
NUMBER OF YEARS? 10
FUTURE VALUE=$ 17388.64

MORE DATA? (1=YES,0=NO)? 1

INITIAL INVESTMENT? 16050
NOMINAL INTEREST RATE? 7
NUMBER OF COMPOUNDING PERIODS PER YEAR? 1
NUMBER OF YEARS? 5.5
FUTURE VALUE=$ 23285.51

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "FUTURE VALUE OF AN INVESTMENT"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "INITIAL INVESTMENT";
 40  INPUT P
 50  PRINT "NOMINAL INTEREST RATE";
```

```
 60  INPUT I
 70  PRINT "NUMBER OF COMPOUNDING PERIODS PER YEAR";
 80  INPUT N
 90  PRINT "NUMBER OF YEARS";
100  INPUT Y
108  REM - CALCULATE INTEREST RATE PER PERIOD;
109  REM - CONVERT FROM PERCENT TO DECIMAL
110  I=I/N/100
119  REM - CALCULATE FUTURE VALUE BY FORMULA
120  T=P*(1+I)↑(N*Y)
129  REM - ROUND OFF TO NEAREST CENT, PRINT
130  PRINT "FUTURE VALUE=$"; INT(T*100+.5)/100
139  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
140  PRINT
149  REM - RESTART OR END PROGRAM? USER INPUT REQUIRED
150  PRINT "MORE DATA? (1=YES,0=NO)";
160  INPUT X
170  IF X=1  THEN   20
180  END
```

OPTION

This program allows you to enter a term of investment in whole years or decimal parts only. In some cases you may wish to enter the term of investment in years and months rather than just years. The program changes necessary follow the example listed below.

Example:

Herb invests $12,000.00 at 8% interest. Interest is compounded quarterly. What is the value of his investment at the end of 10 years and 7 months?

```
;RUN
FUTURE VALUE OF AN INVESTMENT

INITIAL INVESTMENT? 12000
NOMINAL INTEREST RATE? 8
NUMBER OF COMPOUNDING PERIODS PER YEAR? 4
NUMBER OF YEARS, MONTHS? 10,7
FUTURE VALUE=$ 27749.5

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 90-105
 10  PRINT "FUTURE VALUE OF AN INVESTMENT"
  :
 80  INPUT N
 90  PRINT "NUMBER OF YEARS, MONTHS";
100  INPUT Y0,M
104  REM - CALCULATE YEARS FROM YEARS AND MONTHS
105  Y=(12*Y0+M)/12
108  REM - CALCULATE INTEREST RATE PER PERIOD;
  :
180  END
```

Future Value of Regular Deposits (Annuity)

This program calculates a future value when deposits are made regularly. All deposits are equal. You must provide the amount of each deposit, the number of deposits per year, the number of years, and the nominal interest rate.

Assuming that interest is compounded with each deposit, the calculation is based on the following formula:

$$T = R \cdot \left(\frac{(1 + i/N)^{N \cdot Y} - 1}{i/N} \right)$$

where: T = total value after Y years (future value)
R = amount of regular deposits
N = number of deposits per year
Y = number of years
i = nominal interest rate

Examples:

$50.00 is transferred each month from Matt's checking account to a Christmas Club savings account with 5% interest. How much will Matt receive at the end of the year?

Tim makes annuity payments of $175.00. The interest is 5.5%. What amount will Tim have accumulated in 15 years?

```
:RUN
FUTURE VALUE OF REGULAR DEPOSITS (ANNUITY)

AMOUNT OF REGULAR DEPOSITS? 50
NOMINAL INTEREST RATE? 5
NUMBER OF DEPOSITS PER YEAR? 12
NUMBER OF YEARS? 1
FUTURE VALUE = $ 613.94

MORE DATA? (1=YES, 0=NO)? 1

AMOUNT OF REGULAR DEPOSITS? 175
NOMINAL INTEREST RATE? 5.5
NUMBER OF DEPOSITS PER YEAR? 1
NUMBER OF YEARS? 15
FUTURE VALUE = $ 3921.52

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "FUTURE VALUE OF REGULAR DEPOSITS (ANNUITY)"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "AMOUNT OF REGULAR DEPOSITS";
 40  INPUT R
 50  PRINT "NOMINAL INTEREST RATE";
```

```
 60  INPUT I
 70  PRINT "NUMBER OF DEPOSITS PER YEAR";
 80  INPUT N
 90  PRINT "NUMBER OF YEARS";
100  INPUT Y
108  REM - CALCULATE INTEREST RATE PER DEPOSIT,
109  REM - CONVERT FROM PERCENT TO DECIMAL
110  I=I/N/100
119  REM - CALCULATE FUTURE VALUE BY FORMULA
120  T=R*((1+I)↑(N*Y)-1)/I
129  REM - ROUND OFF TO NEAREST CENT, PRINT
130  PRINT "FUTURE VALUE = $"; INT(T*100+.5)/100
139  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
140  PRINT
149  REM - RESTART OR END PROGRAM?  USER INPUT REQUIRED
150  PRINT "MORE DATA? (1=YES, 0=NO)";
160  INPUT X
170  IF X=1  THEN   20
180  END
```

OPTION

You may wish to enter the term of investment in years and months rather than years. The program changes necessary are listed following the example below.

Example:

How much will Ron receive in 10 years and 5 months if he transfers $50.00 each month into a trust fund with 5% interest?

```
:RUN
FUTURE VALUE OF REGULAR DEPOSITS (ANNUITY)

AMOUNT OF REGULAR DEPOSITS? 50
NOMINAL INTEREST RATE? 5
NUMBER OF DEPOSITS PER YEAR? 12
NUMBER OF YEARS, MONTHS? 10,5
FUTURE VALUE = $ 8179.31

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 90-105
 10  PRINT "FUTURE VALUE OF REGULAR DEPOSITS (ANNUITY)"
  :
 80  INPUT N
 90  PRINT "NUMBER OF YEARS, MONTHS";
100  INPUT Y0,M
104  REM - CALCULATE YEARS FROM YEARS AND MONTHS
105  Y=(12*Y0+M)/12
108  REM - CALCULATE INTEREST RATE PER DEPOSIT,
  :
180  END
```

Regular Deposits

This program calculates the amount required as a regular deposit to provide a stated future value in a specified time period. All deposits are equal. It is necessary for you to supply the future value, the nominal interest rate, the number of deposits per year and the number of years.

The calculation for regular deposits is based on the following formula:

$$R = T\left(\frac{i/N}{(1+i/N)^{N\cdot Y}-1}\right)$$

where: R = amount of regular deposit
T = future value
i = nominal interest rate
N = number of deposits per year
Y = number of years

Example:

Mary would like $1000.00 at the end of one year in a savings account. How much must she deposit each month at 8% interest to achieve this?

```
:RUN
REGULAR DEPOSITS

TOTAL VALUE AFTER Y YEARS? 1000
NOMINAL INTEREST RATE? 8
NUMBER OF DEPOSITS PER YEAR? 12
NUMBER OF YEARS? 1
REGULAR DEPOSITS = $ 80.32

MORE DATA?(1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "REGULAR DEPOSITS"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "TOTAL VALUE AFTER Y YEARS";
 40  INPUT T
 50  PRINT "NOMINAL INTEREST RATE";
 60  INPUT I
 70  PRINT "NUMBER OF DEPOSITS PER YEAR";
 80  INPUT N
 90  PRINT "NUMBER OF YEARS";
100  INPUT Y
108  REM - CALCULATE INTEREST RATE PER DEPOSIT;
109  REM - CONVERT FROM PERCENT TO DECIMAL
110  I=I/N/100
```

```
119  REM - CALCULATE AMOUNT OF REGULAR DEPOSIT BY FORMULA.
120  R=T*I/((I+1)↑(N*Y)-1)
129  REM - ROUND OFF TO NEAREST CENT, PRINT
130  PRINT "REGULAR DEPOSITS = $";INT(R*100+.5)/100
139  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
140  PRINT
149  REM - RESTART OR END PROGRAM? USER INPUT REQUIRED
150  PRINT "MORE DATA?(1=YES,0=NO)";
160  INPUT X
170  IF X=1  THEN   20
180  END
```

OPTION

You may wish to enter the term of investment in years and months rather than years. The program changes necessary are listed following the example below.

Example:

Ed would like to save $2000.00 for a new motorcycle. He would like to achieve this amount in 1 year and 5 months. How much must he deposit each month if his interest is 8%?

```
:RUN
REGULAR DEPOSITS

TOTAL VALUE AFTER Y YEARS? 2000
NOMINAL INTEREST RATE? 8
NUMBER OF DEPOSITS PER YEAR? 12
NUMBER OF YEARS, MONTHS? 1,5
REGULAR DEPOSITS = $ 111.5

MORE DATA?(1=YES,0=NO)? 0

END PROGRAM
```

```
PROGRAM LISTING

   1  REM - OPTION 90-105
  10  PRINT "REGULAR DEPOSITS"
   :
  80  INPUT N
  90  PRINT "NUMBER OF YEARS, MONTHS";
 100  INPUT Y0,M
 104  REM - CALCULATE YEARS FROM YEARS AND MONTHS
 105  Y=(12*Y0+M)/12
 108  REM - CALCULATE INTEREST RATE PER DEPOSIT;
   :
 180  END
```

Regular Withdrawals from an Investment

This program calculates the maximum amount which may be withdrawn regularly from an investment over a specified time period. All withdrawals are assumed to be equal. You must provide the amount of the initial investment, the nominal interest rate, the number of withdrawals per year and the number of years.

The maximum amount of withdrawals is calculated by the following formula:

$$R = P\left(\frac{i/N}{(1+i/N)^{N\cdot Y}-1} + \frac{i}{N}\right)$$

where: R = amount of regular withdrawal
P = initial investment
i = nominal interest rate
N = number of withdrawals per year
Y = number of years

Because this program calculates a maximum amount, a balance of $0.00 will be left in your account at the end of the time period. You may withdraw any lesser amount under the same specifications and leave a remaining balance in your account.

Example:

David invests $8000.00 at 9.5%. He plans to make regular withdrawals every month for ten years, leaving nothing at the end. How much should he withdraw each time?

```
:RUN
REGULAR WITHDRAWALS FROM AN INVESTMENT

INITIAL INVESTMENT? 8000
NOMINAL INTEREST RATE? 9.5
NUMBER OF WITHDRAWALS PER YEAR? 12
NUMBER OF YEARS? 10
AMOUNT OF EACH WITHDRAWAL = $ 103.52

MORE DATA?(1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "REGULAR WITHDRAWALS FROM AN INVESTMENT"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "INITIAL INVESTMENT";
 40  INPUT P
 50  PRINT "NOMINAL INTEREST RATE";
 60  INPUT I
 70  PRINT "NUMBER OF WITHDRAWALS PER YEAR";
 80  INPUT N
 90  PRINT "NUMBER OF YEARS";
100  INPUT Y
```

```
108  REM - CALCULATE INTEREST RATE PER WITHDRAWAL;
109  REM - CONVERT FROM PERCENT TO DECIMAL
110  I=I/N/100
119  REM - CALCULATE REGULAR WITHDRAWAL BY FORMULA
120  R=P*(I/((1+I)↑(N*Y)-1)+I)
129  REM - ROUND OFF TO NEAREST CENT, PRINT
130  PRINT "AMOUNT OF EACH WITHDRAWAL = $"; INT(R*100+.5)/100
139  REM - PRINT BLANK LINE TO SEPARATE QUESTION FROM DATA
140  PRINT
149  REM - RESTART OR END PROGRAM?
150  PRINT "MORE DATA?(1=YES,0=NO)";
160  INPUT X
170  IF X=1  THEN   20
180  END
```

OPTION

It may be more convenient to enter the period of investment in years and months rather than just years. The program changes necessary are listed following the example below.

Example:

How much could be withdrawn each week if you have an investment of $8000.00 at 9.5% interest to be withdrawn from for 10 years and 5 months?

```
:RUN
REGULAR WITHDRAWALS FROM AN INVESTMENT

INITIAL INVESTMENT? 8000
NOMINAL INTEREST RATE? 9.5
NUMBER OF WITHDRAWALS PER YEAR? 52
NUMBER OF YEARS, MONTHS? 10,5
AMOUNT OF EACH WITHDRAWAL = $ 23.28

MORE DATA?(1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 90-105
 10  PRINT "REGULAR WITHDRAWALS FROM AN INVESTMENT"
  :
 80  INPUT N
 90  PRINT "NUMBER OF YEARS, MONTHS";
100  INPUT Y0,M
104  REM - CALCULATE YEARS FROM YEARS AND MONTHS
105  Y=(12*Y0+M)/12
108  REM - CALCULATE INTEREST RATE PER WITHDRAWAL;
  :
180  END
```

Initial Investment

This program calculates the investment necessary to provide a stated future value in a specified time period. You must enter the future value of the investment, the number of years of investment, the number of compounding periods per year and the nominal interest rate.

The formula used to calculate the initial investment is as follows:

$$P = \frac{T}{(1 + i/N)^{N \cdot Y}}$$

where: P = initial investment
T = future value
N = number of compounding periods per year
Y = number of years
i = nominal interest rate

Examples:

How much must you invest at 8.5% to produce $10,000.00 at the end of 10 years if interest is compounded quarterly?

Merchant Savings wishes to sell a bond which will be worth $5000.00 five years from the purchase date. Interest will be 7.9% compounded daily. How much must the bank charge for the bond?

```
:RUN
INITIAL INVESTMENT

TOTAL VALUE AFTER Y YEARS? 10000
NUMBER OF COMPOUNDING PERIODS PER YEAR? 4
NUMBER OF YEARS? 10
NOMINAL INTEREST RATE? 8.5
INITIAL INVESTMENT = $ 4312.33

MORE DATA? (1=YES,0=NO)? 1

TOTAL VALUE AFTER Y YEARS? 5000
NUMBER OF COMPOUNDING PERIODS PER YEAR? 365
NUMBER OF YEARS? 5
NOMINAL INTEREST RATE? 7.9
INITIAL INVESTMENT = $ 3368.54

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "INITIAL INVESTMENT"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "TOTAL VALUE AFTER Y YEARS";
 40  INPUT T
```

```
 50  PRINT "NUMBER OF COMPOUNDING PERIODS PER YEAR";
 60  INPUT N
 70  PRINT "NUMBER OF YEARS";
 80  INPUT Y
 90  PRINT "NOMINAL INTEREST RATE";
100  INPUT I
108  REM - CALCULATE INTEREST RATE PER PERIOD;
109  REM - CONVERT FROM % TO DECIMAL
110  I=I/N/100
119  REM - CALCULATE INITIAL INVESTMENT BY FORMULA
120  P=T/(1+I)↑(N*Y)
129  REM - ROUND OFF TO NEAREST CENT, PRINT
130  PRINT "INITIAL INVESTMENT = $";INT(P*100+.5)/100
139  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
140  PRINT
149  REM - RESTART OR END PROGRAM? USER INPUT REQUIRED
150  PRINT "MORE DATA? (1=YES,0=NO)";
160  INPUT X
170  IF X=1  THEN   20
180  END
```

OPTION

The program above allows you to enter a period of investment of whole years and decimal parts only. You may wish to enter the period of investment in years and months rather than just years. The program changes necessary are listed following the example below.

Example:

Mary wishes to invest a sum in a savings bank. In 3 years and 8 months she would like to have $4000.00 in her account. If 8% interest is compounded monthly, what amount must Mary invest?

```
:RUN
INITIAL INVESTMENT

TOTAL VALUE AFTER Y YEARS? 4000
NUMBER OF COMPOUNDING PERIODS PER YEAR? 12
NUMBER OF YEARS, MONTHS? 3,8
NOMINAL INTEREST RATE? 8
INITIAL INVESTMENT = $ 2986

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 70-85
 10  PRINT "INITIAL INVESTMENT"
  :
 60  INPUT N
 70  PRINT "NUMBER OF YEARS, MONTHS";
 80  INPUT Y0,M
 84  REM - CALCULATE YEARS FROM YEARS AND MONTHS
 85  Y=(12*Y0+M)/12
 90  PRINT "NOMINAL INTEREST RATE";
  :
180  END
```

Minimum Investment for Withdrawals

This program calculates the minimum investment required to allow regular withdrawals over a specified time period. The amount calculated is dependent upon the amount of each withdrawal, the number of withdrawals per year, the number of years, and the nominal interest rate on the investment. All withdrawals are equal.

Only the least amount necessary for your investment is calculated; the program assumes a balance of $0.00 to be left at the end of the time period. Any investment larger than the amount calculated will also enable you to withdraw the desired amount, but leave a remaining balance.

Assuming that interest is compounded with each withdrawal, the calculation is based on the following formula:

$$P = \frac{R \cdot N}{i} \left(1 - \frac{1}{(1 + i/N)^{N \cdot Y}} \right)$$

where: P = initial investment
R = amount of regular withdrawal
i = nominal interest rate
N = number of withdrawals per year
Y = number of years

Example:

How much must you invest at 6% interest to allow monthly withdrawals of $100.00 for 5 years?

```
:RUN
MINIMUM INVESTMENT FOR WITHDRAWALS

AMOUNT OF WITHDRAWALS? 100
NOMINAL INTEREST RATE? 6
NUMBER OF WITHDRAWALS PER YEAR? 12
NUMBER OF YEARS? 5
MINIMUM INVESTMENT = $ 5172.55

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "MINIMUM INVESTMENT FOR WITHDRAWALS"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "AMOUNT OF WITHDRAWALS";
 40  INPUT R
 50  PRINT "NOMINAL INTEREST RATE";
 60  INPUT I
 70  PRINT "NUMBER OF WITHDRAWALS PER YEAR";
 80  INPUT N
 90  PRINT "NUMBER OF YEARS";
100  INPUT Y
```

```
109  REM - CONVERT FROM PERCENT TO DECIMAL
110  I=I/100
119  REM - CALCULATE MINIMUM INVESTMENT BY FORMULA
120  P=R*N/I*(1-1/((1+I/N)↑(N*Y)))
129  REM - ROUND OFF TO NEAREST CENT, PRINT
130  PRINT "MINIMUM INVESTMENT = $";INT(100*P+.5)/100
139  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
140  PRINT
149  REM - RESTART OR END PROGRAM? USER INPUT REQUIRED
150  PRINT "MORE DATA (1=YES, 0=NO)";
160  INPUT X
170  IF X=1  THEN   20
180  END
```

OPTION

It may be more convenient to enter the term of investment in years and months rather than years. The program changes necessary are listed following the example below.

Example:

Tony withdrew $250.00 monthly for 6 years and 5 months. How much was his initial investment at 6% interest?

```
:RUN
MINIMUM INVESTMENT FOR WITHDRAWALS

AMOUNT OF WITHDRAWALS? 250
NOMINAL INTEREST RATE? 6
NUMBER OF WITHDRAWALS PER YEAR? 12
NUMBER OF YEARS, MONTHS? 6,5
MINIMUM INVESTMENT = $ 15944.81

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 90-105
 10  PRINT "MINIMUM INVESTMENT FOR WITHDRAWALS"
  :
 80  INPUT N
 90  PRINT "NUMBER OF YEARS, MONTHS";
100  INPUT Y0,M
104  REM - CALCULATE YEARS FROM YEARS AND MONTHS
105  Y=(12*Y0+M)/12
109  REM - CONVERT FROM PERCENT TO DECIMAL
  :
180  END
```

Nominal Interest Rate on Investments

This program calculates the nominal interest rate for a known initial investment which amounts to a known future value in a specified period of time. The nominal interest rate is usually subdivided for compounding purposes.

"Nominal Interest Rate" is based on the following formula:

$$i = N(T/P)^{\frac{1}{N \cdot Y}} - N$$

where: i = nominal interest rate
P = initial investment
T = future value
N = number of compounding periods per year
Y = number of years

The nominal interest rate is expressed as a yearly rate even though the interest rate used when compounding interest is $\frac{i}{N}$. The nominal interest rate will be less than the effective interest rate when interest is compounded more than once a year. This is because the nominal rate stated does not take into account interest compounded on interest earned in earlier periods of each year. For example, the schedule of earned interest on $100.00 at 5% compounded quarterly would be:

PERIOD	BALANCE	$\frac{i/100}{N}$		INTEREST	NEW BALANCE
1	$100.00	• .0125	=	$1.25	$101.25
2	$101.25	• .0125	=	$1.27	$102.52
3	$102.52	• .0125	=	$1.28	$103.80
4	$103.80	• .0125	=	$1.30	$105.10

The *effective* interest rate in the example is 5.1%, although the *nominal* rate is 5%.

Examples:

Jane invests $945.00 in a savings bank. Four and a half years later her investment amounts to $1309.79. If interest is compounded monthly, what is the nominal interest rate offered by the bank?

Dick invests $3,000.00. Ten years later he has earned $1,576.00 in interest. If interest is compounded each month, what is the nominal interest rate on the account?

```
:RUN
NOMINAL INTEREST RATE ON INVESTMENTS

PRINCIPAL? 945
TOTAL VALUE? 1309.79
NUMBER OF YEARS? 4.5
NUMBER OF COMPOUNDING PERIODS PER YEAR? 12
NOMINAL INTEREST RATE= 7.2761298 %

MORE DATA: (1=YES,0=NO)? 1

PRINCIPAL? 3000
TOTAL VALUE? 4576
NUMBER OF YEARS? 10
```

```
NUMBER OF COMPOUNDING PERIODS PER YEAR? 12
NOMINAL INTEREST RATE= 4.22956608 %

MORE DATA: (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "NOMINAL INTEREST RATE ON INVESTMENTS"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "PRINCIPAL";
 40  INPUT P
 50  PRINT "TOTAL VALUE";
 60  INPUT T
 70  PRINT "NUMBER OF YEARS";
 80  INPUT Y
 90  PRINT "NUMBER OF COMPOUNDING PERIODS PER YEAR";
100  INPUT N
109  REM - CALCULATE NOMINAL INTEREST RATE BY FORMULA, PRINT
110  I2=N*((T/P)↑(1/(N*Y))-1)*100
120  PRINT "NOMINAL INTEREST RATE=";I2;"%"
129  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
130  PRINT
139  REM - RESTART OR END PROGRAM? USER INPUT REQUIRED
140  PRINT "MORE DATA: (1=YES,0=NO)";
150  INPUT X
160  IF X=1  THEN   20
170  END
```

Effective Interest Rate on Investments

This program calculates the effective interest rate for a known initial investment which amounts to a known future value in a specified period of time. This rate expresses the actual rate of interest earned annually on the investment.

The effective interest rate is calculated by the following formula:

$$\text{effective interest rate} = \left(\frac{\text{future value}}{\text{initial investment}}\right)^{1/\text{years}} - 1$$

You may calculate the effective interest rate on amounts you have already invested and accrued interest. Or you may calculate the effective interest rate necessary to enable a principal to reach a hypothetical value in a specified amount of time. For instance, if you invest $5000.00 in a bank and desire $6800.00 after six years, you will predict the effective interest rate the bank must pay in order to achieve this.

"Effective Interest Rate" may also be used to calculate the effective percent of depreciation of an investment. Take your car, for example. If you bought it for $7534.00 and sold it for $3555.00 three years later, you will find that its actual depreciation (a negative interest rate) was approximately 22% each year.

Examples:

Jane deposits $945.00 in a savings bank. Four and a half years later her account has $1309.79. What actual percent of her initial investment did the bank pay annually?

Dick bought his car in 1970 for $7534.84 and sold it in 1973 for $3555.00. What was its effective rate of depreciation?

```
:RUN
EFFECTIVE INTEREST RATE ON INVESTMENTS

INITIAL INVESTMENT? 945
TOTAL VALUE AFTER Y YEARS? 1309.79
NUMBER OF YEARS? 4.5
ANNUAL INTEREST RATE = 7.5237528 %

MORE DATA? (1=YES,0=NO)? 1

INITIAL INVESTMENT? 7534.84
TOTAL VALUE AFTER Y YEARS? 3555
NUMBER OF YEARS? 3
ANNUAL INTEREST RATE =-22.150614266 %

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "EFFECTIVE INTEREST RATE ON INVESTMENTS"
 20  PRINT
 29  REM - STATEMENTS 30-80 REQUEST USER INPUT
 30  PRINT "INITIAL INVESTMENT";
 40  INPUT P
 50  PRINT "TOTAL VALUE AFTER Y YEARS";
 60  INPUT T
 70  PRINT "NUMBER OF YEARS";
 80  INPUT Y
 89  REM - CALCULATE EFFECTIVE INTEREST RATE, PRINT
 90  PRINT "ANNUAL INTEREST RATE =";((T/P)↑(1/Y)-1)*100;"%"
 99  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
100  PRINT
109  REM - RESTART OR END PROGRAM?
110  PRINT "MORE DATA? (1=YES,0=NO)";
120  INPUT X
130  IF X=1  THEN   20
140  END
```

Earned Interest Table

This program calculates and prints an earned interest table for investments. The schedule contains the following outputs:

1) Periodic balance
2) Interest accumulated between two periods
3) Total interest accumulated
4) Effective interest rate

These cutputs may be calculated for a single investment or for an initial investment with regular deposits or withdrawals. If the table is to be tabulated for a single investment, you must provide the amount of the initial investment, the nominal interest rate, and the number of compounding periods per year. Your new balance will be printed a maximum of four times per year. If interest is compounded less than four times per year, your new balance will be posted with each interest computation.

If the table is tabulated for regular deposits or withdrawals, you must provide the amount of the initial investment, the nominal interest rate, the number of deposits or withdrawals per year and their amount. In this case it is assumed interest is compounded daily (360-day year). Your new balance will be printed at each deposit or withdrawal.

Examples:

Sally invests $2000.00 at 9.5% in a trust fund for ten years. Interest is compounded monthly. What is her yearly balance and earned interest for the last two years?

John deposits $1000.00 at 8% in a passbook savings account. From each monthly paycheck $50.00 is deposited in this account. What is the earned interest table for the first year of this account?

Ted deposits $1000.00 at 8% in his savings. Each quarter he withdraws $150.00. What is the earned interest table for year one of this account?

```
:RUN
EARNED INTEREST TABLE

PRINCIPAL? 2000
NOMINAL INTEREST RATE? 9.5
NUMBER OF DEPOSITS/WITHDRAWALS PER YEAR? 0
NUMBER OF COMPOUNDING PERIODS PER YEAR? 12
START WITH WHAT YEAR? 9
END PRINTING WITH WHAT YEAR? 10
```

```
                    EARNED INTEREST TABLE
  PRINCIPAL $ 2000      AT 9.5 % NOMINAL      FOR 10 YEARS
            EFFECTIVE INTEREST RATE 9.92 % PER YEAR

YEAR          BALANCE          INTEREST          ACCUM.INTEREST

 9             4365.87          2365.86           2365.87
               4470.38          104.51            2470.38
               4577.39          107.01            2577.39
               4686.97          109.58            2686.97

 10            4799.17          112.2             2799.17
               4914.06          114.89            2914.06
               5031.7           117.64            3031.7
               5152.15          120.45            3152.15

CHANGE DATA AND RECOMPUTE? (1=YES, 0=NO)? 1


PRINCIPAL? 1000
NOMINAL INTEREST RATE? 8
NUMBER OF DEPOSITS/WITHDRAWALS PER YEAR? 12
AMOUNT OF DEPOSIT/WITHDRAWAL? 50
START WITH WHAT YEAR? 1
END PRINTING WITH WHAT YEAR? 1

                    EARNED INTEREST TABLE
  PRINCIPAL $ 1000      AT 8 % NOMINAL      FOR 1 YEARS
REGULAR DEPOSIT/WITHDRAWAL $ 50      12  TIMES PER YEAR
            EFFECTIVE INTEREST RATE 8.33 % PER YEAR

YEAR          BALANCE          INTEREST          ACCUM.INTEREST

 1             1056.7           6.7               6.7
               1113.78          7.08              13.78
               1171.24          7.46              21.24
               1229.08          7.84              29.08
               1287.32          8.23              37.32
               1345.94          8.62              45.94
               1404.95          9.01              54.95
               1464.36          9.41              64.36
               1524.17          9.81              74.17
               1584.38          10.21             84.38
               1644.98          10.61             94.98
               1706             11.01             106

CHANGE DATA AND RECOMPUTE? (1=YES, 0=NO)? 1


PRINCIPAL? 1000
NOMINAL INTEREST RATE? 8
NUMBER OF DEPOSITS/WITHDRAWALS PER YEAR? 4
AMOUNT OF DEPOSIT/WITHDRAWAL? -150
START WITH WHAT YEAR? 1
END PRINTING WITH WHAT YEAR? 1
```

```
                    EARNED INTEREST TABLE
  PRINCIPAL $ 1000       AT 8 % NOMINAL      FOR 1 YEARS
REGULAR DEPOSIT/WITHDRAWAL $-150       4  TIMES PER YEAR
             EFFECTIVE INTEREST RATE 8.33 % PER YEAR

YEAR              BALANCE          INTEREST         ACCUM.INTEREST

 1                 870.17            20.17             20.17
                   737.71            17.54             37.71
                   602.58            14.87             52.58
                   464.72            12.14             64.72

CHANGE DATA AND RECOMPUTE? (1=YES, 0=NO)? 0


END PROGRAM


PROGRAM LISTING

  10  PRINT "EARNED INTEREST TABLE"
  20  PRINT
  29  REM - STATEMENTS 30 TO 230 REQUEST USER INPUT
  30  PRINT "PRINCIPAL";
  40  INPUT P
  50  PRINT "NOMINAL INTEREST RATE";
  60  INPUT I
  69  REM - CONVERT PERCENT TO DECIMAL
  70  I=I/100
  80  PRINT  "NUMBER OF DEPOSITS/WITHDRAWALS PER YEAR";
  90  INPUT N1
  99  REM - DON'T ASK FOR AMOUNT IF FREQUENCY IS ZERO
 100  IF N1=0  THEN  160
 108  REM - DEPOSITS ARE ENTERED AS A POSITIVE NUMBER
 109  REM - WITHDRAWALS ARE ENTERED AS A NEGATIVE NUMBER
 110  PRINT  "AMOUNT OF DEPOSIT/WITHDRAWAL";
 120  INPUT R
 129  REM - INTEREST IS COMPOUNDED DAILY
 130  N=360
 139  REM - PRINT AT EACH DEPOSIT/WITHDRAWAL
 140  L2=N1
 150  GOTO  200
 160  PRINT "NUMBER OF COMPOUNDING PERIODS PER YEAR";
 170  INPUT N
 180  N1=0
 189  REM - PRINT FOUR TIMES PER YEAR
 190  L2=4
 200  PRINT "START WITH WHAT YEAR";
 210  INPUT X
 220  PRINT "END PRINTING WITH WHAT YEAR";
 230  INPUT Y
 239  REM - START PRINTING AT THE BEGINNING OF A YEAR
 240  X=INT(X)
 249  REM - INITIATE RUNNING TOTALS
 250  B0=P
 260  I1=0
 270  I2=0
```

```
280  I3=0
290  K=66
300  P1=4
310  FOR J0=1  TO INT(Y) +1
319  REM - START PRINTING?
320  IF J0<X  THEN  480
329  REM - TEST FOR END OF PAGE
330  IF K<55  THEN  470
339  REM - SPACE TO NEXT PAGE, PRINT HEADINGS (ASSUMED 66 LINES PER PAGE)
340  FOR K1=K  TO 66
350  PRINT
360  NEXT K1
370  K=6
380  PRINT "                              EARNED INTEREST TABLE"
390  PRINT "  PRINCIPAL $";P;"   AT";I*100;"% NOMINAL   FOR";Y;"YEARS
   "
399  REM - SKIP DEPOSIT/WITHDRAWAL HEADING IF THERE ARE NONE
400  IF N1=0  THEN  430
410  PRINT "REGULAR DEPOSIT/WITHDRAWAL $";R;"  ";N1;" TIMES PER YEAR"
419  REM - K COUNTS THE NUMBER OF PRINTED LINES PER PAGE
420  K=K+1
430  PRINT "          EFFECTIVE INTEREST RATE";FNR(100*((1+I/N)↑N-1));
   "% PER YEAR"
440  PRINT
450  PRINT "YEAR","BALANCE","INTEREST","ACCUM.INTEREST"
459  REM - CALCULATE INTEREST
460  PRINT
469  REM - PRINT YEAR NUMBER
470  PRINT J0,
480  L1=1
490  N2=1
500  P2=1
510  FOR J1=1  TO N
519  REM - DEPOSIT/WITHDRAW ANY MORE THIS YEAR?
520  IF N2>N1  THEN  560
529  REM - TIME TO MAKE DEPOSIT/WITHDRAWAL?
530  IF N2/N1>J1/N  THEN  560
539  REM - CALCULATE NEW BALANCE
540  B0=B0+R
549  REM - COUNT DEPOSITS/WITHDRAWALS MADE PER YEAR
550  N2=N2+1
560  B2=B0*(1+I/N)
569  REM - I1=AMOUNT INTEREST WITH EACH COMPOUNDING PERIOD
570  I1=B2-B0
579  REM - I3=AMOUNT INTEREST ACCUMULATED BETWEEN POSTING
580  I3=I3+I1
589  REM - I2=TOTAL INTEREST ACCUMULATED TO DATE
590  I2=I2+I1
599  REM - ROUND AT INTEREST POSTING TIME
600  IF P2/P1>J1/N  THEN  640
610  I2=FNR(I2)
620  B2=FNR(B2)
630  P2=P2+1
639  REM - YEAR TO START PRINTING?
640  IF J0<X  THEN  710
649  REM - TIME TO PRINT A LINE?
650  IF J1/N<L1/L2  THEN  710
```

```
660  L1=L1+1
670  PRINT FNR(B2),FNR(I3),FNR(I2)
679  REM - INTEREST POSTED, REINITIALIZE INTEREST ACCUM. BETWEEN POSTINGS
680  I3=0
690  K=K+1
699  REM - YEAR NUMBER PRINTED WITH FIRST POSTING IN EACH YEAR ONLY
700  PRINT
710  B0=B2
719  REM -- NO MORE LINES TO PRINT IN LAST YEAR?
720  IF J0+J1/N-1>=Y  THEN  780
730  NEXT J1
739  REM - START PRINTING?
740  IF J0<X  THEN  770
750  PRINT
760  K=K+1
770  NEXT J0
780  PRINT
789  REM - RESTART OR END PROGRAM?
790  PRINT "CHANGE DATA AND RECOMPUTE?. (1=YES, 0=NO)";
800  INPUT Z
810  PRINT
820  IF Z=1  THEN   20
829  REM - ROUND OFF FUNCTION
830  DEFFNR(X)=INT(X*100+.5)/100
840  END
```

Depreciation Rate

This program calculates the annual depreciation rate of an investment. You must provide the original price of the item, its resale price, and its age in years.

The depreciation rate is calculated by the following formula:

$$\text{depreciation rate} = 1 - \left(\frac{\text{resale price}}{\text{original price}}\right)^{1/\text{age}}$$

Example:

Joan bought her car for $4933.76 and sold it for $2400.00 three years later. What was its actual depreciation rate?

```
:RUN
DEPRECIATION RATE

ORIGINAL PRICE? 4933.76
RESALE PRICE? 2400
YEARS? 3
DEPRECIATION RATE = 21.354 %

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "DEPRECIATION RATE"
 20  PRINT
 30  PRINT "ORIGINAL PRICE";
 40  INPUT P
 50  PRINT "RESALE PRICE";
 60  INPUT T
 70  PRINT "YEARS";
 80  INPUT Y
 89  REM - CALCULATE DEPRECIATION RATE BY FORMULA, CONVERT TO PERCENT
 90  D=100*(1-(T/P)↑(1/Y))
 99  REM - ROUND OFF, PRINT
100  PRINT "DEPRECIATION RATE =";INT(1000*D+.5)/1000;"%"
110  PRINT
119  REM - RESTART OR END PROGRAM?
120  PRINT "MORE DATA (1=YES, 0=NO)";
130  INPUT X
140  IF X=1  THEN   20
150  END
```

Depreciation Amount

This program calculates the dollar amount depreciated within a given year for a depreciating investment. You must provide the original price of the investment, its depreciation rate, and the year of depreciation.

The depreciation amount is calculated by the following formula:

$$D = P \cdot i \cdot (1 - i)^{Y - 1}$$

where: D = depreciation amount
P = original price
i = depreciation rate
Y = year of depreciation

Examples:

Joan bought her car for $4933.76. Her model car depreciates at an average annual rate of 21%. What amount has the car depreciated in each of the first three years she has owned it?

Joan is also concerned about the depreciation of the tape deck in her car. It cost her $155.00 two years ago, and has a depreciation rate of 22%. How much will its value decline in year three?

```
:RUN
DEPRECIATION AMOUNT

ORGINAL PRICE? 4933.76
DEPRECIATION RATE? 21
--(ENTER YEAR=0 WHEN NO MORE AMOUNTS DESIRED FOR THIS ITEM)--
YEAR? 1
DEPRECIATION = $ 1036.09

YEAR? 2
DEPRECIATION = $ 818.51

YEAR? 3
DEPRECIATION = $ 646.62

YEAR? 0
MORE DATA (1=YES, 0=NO)? 1

ORGINAL PRICE? 155
DEPRECIATION RATE? 22
--(ENTER YEAR=0 WHEN NO MORE AMOUNTS DESIRED FOR THIS ITEM)--
YEAR? 3
DEPRECIATION = $ 20.75

YEAR? 0
MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "DEPRECIATION AMOUNT"
 20  PRINT
 30  PRINT "ORGINAL PRICE";
 40  INPUT P
 50  PRINT "DEPRECIATION RATE";
 60  INPUT I
 69  REM - CONVERT FROM PERCENT TO DECIMAL
 70  I=I/100
 80  PRINT "--(ENTER YEAR=0 WHEN NO MORE AMOUNTS DESIRED FOR THIS ITEM
   )--"
 90  PRINT "YEAR";
100  INPUT Y
109  REM - THROUGH CALCULATING FOR THIS ITEM?
110  IF Y=0  THEN  160
119  REM - CALCULATE DEPRECIATION AMOUNT BY FORMULA
120  D=P*I*(1-I)↑(Y-1)
129  REM - ROUND OFF TO NEAREST CENT, PRINT
130  PRINT "DEPRECIATION = $";INT(D*100+.5)/100
140  PRINT
149  REM - RETURN FOR NEXT YEAR NUMBER
150  GOTO   90
159  REM - RESTART OR END PROGRAM?
160  PRINT "MORE DATA (1=YES, 0=NO)";
170  INPUT X
180  IF X=1  THEN   20
190  END
```

Salvage Value

This program calculates the salvage value of an item at the end of a given year. It is necessary for you to provide the age of the item, its original price, and its depreciation rate.

The salvage value is obtained by the following formula:

$$S = P(1 - i)^Y$$

where: S = salvage value
P = original price
i = depreciation rate
Y = age in years

Example:

What is the salvage value of Joan's car if it is three years old, she bought it for $4933.76, and it depreciates 21% annually? What would its salvage value be next year?

Joan's tape deck is 2 years old. What is its value if it cost $155.00 originally and depreciates at a rate of 22%?

```
:RUN
SALVAGE VALUE

ORIGINAL PRICE? 4933.76
DEPRECIATION RATE? 21
--(ENTER YEARS=0 WHEN NO MORE VALUES DESIRED FOR THIS ITEM)--
YEARS? 3
VALUE = $ 2432.54

YEARS? 4
VALUE = $ 1921.7

YEARS? 0
MORE DATA (1=YES, 0=NO)? 1

ORIGINAL PRICE? 155
DEPRECIATION RATE? 22
--(ENTER YEARS=0 WHEN NO MORE VALUES DESIRED FOR THIS ITEM)--
YEARS? 2
VALUE = $ 94.3

YEARS? 0
MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
10  PRINT "SALVAGE VALUE"
20  PRINT
30  PRINT "ORIGINAL PRICE";
```

```
 40  INPUT P
 50  PRINT "DEPRECIATION RATE";
 60  INPUT I
 70  PRINT "--(ENTER YEARS=0 WHEN NO MORE VALUES DESIRED FOR THIS ITEM
   )--"
 80  PRINT "YEARS";
 90  INPUT Y
 99  REM - CALCULATE ANOTHER SALVAGE VALUE?
100  IF Y=0  THEN  140
108  REM - CALCULATE SALVAGE VALUE BY FORMULA, ROUND OFF, PRINT
109  REM - DEPRECIATION RATE CONVERTED TO DECIMAL FOR USE IN CALCULATI
   ONS
110  PRINT "VALUE = $";INT(100*P*(1-I/100)↑Y+.5)/100
120  PRINT
129  REM - RETURN FOR NEXT YEAR NUMBER
130  GOTO   80
139  REM - RESTART OR END PROGRAM?
140  PRINT "MORE DATA (1=YES, 0=NO)";
150  INPUT X
160  IF X=1  THEN   20
170  END
```

Discount Commercial Paper

This program calculates the amount of discount and net cost of a discounted commercial paper. You must provide the future value of the paper, the discount rate and the number of days to maturity.

The formulas used to calculate the discount and cost are as follows:

$$\text{discount} = T \cdot \frac{D}{100} \cdot \frac{N}{360}$$

$$\text{cost} = T - \text{discount}$$

where: T = total future value
D = discount rate
N = number of days to maturity

Example:

Canning Corporation purchases a $625,000.00 commercial paper due in 60 days at 5.4%. What is the discount and cost?

```
:RUN
DISCOUNT COMMERCIAL PAPER

FUTURE VALUE? 625000
DISCOUNT RATE? 5.4
DAYS TO MATURITY? 60
DISCOUNT = $ 5625
    COST = $ 619375

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "DISCOUNT COMMERCIAL PAPER"
 20  PRINT
 29  REM - STATEMENTS 30 TO 90 REQUEST USER INPUT
 30  PRINT "FUTURE VALUE";
 40  INPUT T
 50  PRINT "DISCOUNT RATE";
 60  INPUT D
 69  REM - CONVERT PERCENT TO DECIMAL
 70  D=D/100
 80  PRINT "DAYS TO MATURITY";
 90  INPUT N
 99  REM - CALCULATE DISCOUNT, PRINT
100  D1=T*D*N/360
110  PRINT "DISCOUNT = $";D1
119  REM - CALCULATE COST, PRINT
120  PRINT "    COST = $";T-D1
129  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
```

```
130   PRINT
139   REM - RESTART OR END PROGRAM? USER INPUT REQUIRED
140   PRINT "MORE DATA (1=YES, 0-NO)";
150   INPUT X
160   IF X=1   THEN    20
170   END
```

Principal on a Loan

This program calculates an initial amount borrowed. This amount is dependent upon the interest rate, the amount of regular payments, the number of payments per year and the term of the loan.

The calculation is based on the formula:

$$P = \frac{R \cdot N}{i} \cdot \left(1 - \frac{1}{(1+i/N)^{N \cdot Y}}\right)$$

where: P = principal
R = regular payment
i = annual interest rate
N = number of payments per year
Y = number of years

Example:

Susan has agreed to pay $250.00 bimonthly for 3 years to repay a loan with 20% interest. What is the amount of the loan?

Tom can afford to make payments of $180.00 per month to repay a loan. If he is willing to make payments for four and a half years and the loan company charges 16% interest, what is the maximum amount Tom can borrow?

```
:RUN
PRINCIPAL ON A LOAN

REGULAR PAYMENT? 250
TERM IN YEARS? 3
ANNUAL INTEREST RATE? 20
NUMBER OF PAYMENTS PER YEAR? 6
PRINCIPAL = $ 3343.45

MORE DATA? (1=YES, 0=NO)? 1

REGULAR PAYMENT? 180
TERM IN YEARS? 4.5
ANNUAL INTEREST RATE? 16
NUMBER OF PAYMENTS PER YEAR? 12
PRINCIPAL = $ 6897.51

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT  "PRINCIPAL ON A LOAN"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "REGULAR PAYMENT";
```

```
 40  INPUT R
 50  PRINT  "TERM IN YEARS";
 60  INPUT Y
 70  PRINT  "ANNUAL INTEREST RATE";
 80  INPUT I
 90  PRINT  "NUMBER OF PAYMENTS PER YEAR";
100  INPUT N
108  REM - CALCULATE AMOUNT OF PRINCIPAL BY FORMULA;
109  REM - INTEREST CONVERTED FROM PERCENT TO DECIMAL FOR CALCULATIONS
110  P=R*N*(1-1/((I/100)/N+1)↑(N*Y))/(I/100)
119  REM - ROUND OFF TO NEAREST CENT, PRINT
120  PRINT "PRINCIPAL = $";INT(P*100+.5)/100
129  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
130  PRINT
139  REM - RESTART OR END PROGRAM?
140  PRINT  "MORE DATA? (1=YES, 0=NO)";
150  INPUT X
160  IF X=1  THEN   20
170  END
```

OPTION

In some cases it may be more convenient to enter the term of the loan in years and months rather than just years. The program changes necessary are listed following the example below.

Example:

What would be the amount of the mortgage if you were paying $75.00 a month for 11 months with 3% interest?

```
:RUN
PRINCIPAL ON A LOAN

REGULAR PAYMENT? 75
TERM IN YEARS, MONTHS? 0,11
ANNUAL INTEREST RATE? 3
NUMBER OF PAYMENTS PER YEAR? 12
PRINCIPAL = $ 812.76

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 50-65
 10  PRINT  "PRINCIPAL ON A LOAN"
  :
 40  INPUT R
 50  PRINT  "TERM IN YEARS, MONTHS";
 60  INPUT Y0,M
 64  REM - CALCULATE YEARS FROM YEARS AND MONTHS
 65  Y=(12*Y0+M)/12
 70  PRINT  "ANNUAL INTEREST RATE";
  :
170  END
```

Regular Payment on a Loan

This program calculates the amount required as regular payments in order to repay a loan over a specified time period. The specifications you must provide are the amount of the principal, the interest rate charged, the number of payments to be made per year and the number of years to pay. This program assumes all installment payments will be equal.

The calculation is based on the formula:

$$R = \frac{i \cdot P/N}{1-\left(\frac{i}{N}+1\right)^{-N \cdot Y}}$$

where: R = regular payment
i = annual interest rate
P = principal
N = number of payments per year
Y = number of years

Examples:

What must you pay on a loan of $4000.00 at 8% if payments are to be made quarterly for five years?

If Michael borrows $6500.00 at 12.5% from Best Rate Savings & Loan to be paid back over a period of 5.5 years, what would his monthly payments be?

```
:RUN
REGULAR PAYMENT ON A LOAN

TERM IN YEARS? 5
PRINCIPAL? 4000
ANNUAL INTEREST RATE? 8
NUMBER OF PAYMENTS PER YEAR? 4
REGULAR PAYMENT = $ 244.63

MORE DATA? (1=YES, 0=NO)? 1

TERM IN YEARS? 5.5
PRINCIPAL? 6500
ANNUAL INTEREST RATE? 12.5
NUMBER OF PAYMENTS PER YEAR? 12
REGULAR PAYMENT = $ 136.68

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
10  PRINT "REGULAR PAYMENT ON A LOAN"
20  PRINT
29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
```

```
 30  PRINT "TERM IN YEARS";
 40  INPUT Y
 50  PRINT "PRINCIPAL";
 60  INPUT P
 70  PRINT "ANNUAL INTEREST RATE";
 80  INPUT I
 90  PRINT "NUMBER OF PAYMENTS PER YEAR";
100  INPUT N
108  REM - CALCULATE AMOUNT OF REGULAR PAYMENT BY FORMULA;
109  REM - INTEREST CONVERTED FROM PERCENT TO DECIMAL FOR CALCULATIONS
110  R=((I/100)*P/N)/(1-1/((I/100)/N+1)↑(N*Y))
119  REM - ROUND OFF TO NEAREST CENT, PRINT
120  PRINT "REGULAR PAYMENT = $";INT(R*100+.5)/100
129  REM - PRINT BLANK LINE TO SEPARATE DATA FROM QUESTION
130  PRINT
139  REM - RESTART OR END PROGRAM?
140  PRINT "MORE DATA? (1=YES, 0=NO)";
150  INPUT X
160  IF X=1  THEN   20
170  END
```

OPTION

You may find it more convenient to enter the term of payment in years and months rather than years. The program changes necessary are listed following the example below.

Example:

Mr. Terry needs $10,000.00 to put down on a new home. Best Rates offers this amount at 14.0% interest to be repaid over a period of 11 years and 5 months. What would be the amount of regular monthly payments?

```
:RUN
REGULAR PAYMENT ON A LOAN

TERM IN YEARS, MONTHS? 11,5
PRINCIPAL? 10000
ANNUAL INTEREST RATE? 14
NUMBER OF PAYMENTS PER YEAR? 12
REGULAR PAYMENT = $ 146.59

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 30-45
 10  PRINT "REGULAR PAYMENT ON A LOAN"
  :
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "TERM IN YEARS, MONTHS";
 40  INPUT Y0,M
 44  REM - CALCULATE YEARS FROM YEARS AND MONTHS
 45  Y=(12*Y0+M)/12
 50  PRINT "PRINCIPAL";
  :
170  END
```

Last Payment on a Loan

This program calculates the amount of the final payment on a loan. This final payment will complete amortization of a loan at the conclusion of its term. You must provide the amount of the loan, the amount of the regular payment, the interest rate charged, the number of payments per year and the term of payment.

The amount of the last payment is normally different from the amount of the regular payment. The final payment will be a "balloon" payment if the final payment is larger than the regular payment. A balloon payment is necessary if applying the amount of the regular payment as the last payment leaves a remaining balance due. In order to entirely pay off the loan at the end of its term, this remaining balance is added to the amount of the regular payment to determine the amount of the last payment.

On the other hand, the amount of the final payment is sometimes less than the regular payment. If the regular payment as the last payment would result in a negative loan balance, then the last payment should be smaller. In this case the regular payment is adjusted by the amount of this hypothetical negative balance to determine the amount of the last payment.

amount of last payment = regular payment + hypothetical balance due on a loan after $N \cdot Y$ regular payments

where: N = number of payments per year
Y = number of years

Examples:

Lynn borrowed $6000.00 at 5% from her father for college expenses. If she pays $1000.00 annually for seven years, what will her last payment be?

Lynn borrows $1150.00 at 8% interest to be repaid at a rate of $75.00 per month. A year and two months later Lynn decides to go to Europe. How much must she pay next month to completely pay off her loan?

```
:RUN
LAST PAYMENT ON A LOAN

REGULAR PAYMENT? 1000
PRINCIPAL? 6000
TERM IN YEARS? 7
ANNUAL INTEREST RATE? 5
NUMBER OF PAYMENTS PER YEAR? 1
LAST PAYMENT = $ 1300.59

MORE DATA? (1=YES,0=NO)? 1

REGULAR PAYMENT? 75
PRINCIPAL? 1150
TERM IN YEARS? 1.17
ANNUAL INTEREST RATE? 8
NUMBER OF PAYMENTS PER YEAR? 12
LAST PAYMENT = $ 240.39

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT  "LAST PAYMENT ON A LOAN"
 20  PRINT
 29  REM - STATEMENTS 30 TO 130 REQUEST USER INPUT
 30  PRINT  "REGULAR PAYMENT";
 40  INPUT R
 50  PRINT  "PRINCIPAL";
 60  INPUT P
 70  PRINT  "TERM IN YEARS";
 80  INPUT Y
 90  PRINT  "ANNUAL INTEREST RATE";
100  INPUT I
109  REM - CONVERT INTEREST FROM PERCENT TO DECIMAL
110  I=I/100
120  PRINT "NUMBER OF PAYMENTS PER YEAR";
130  INPUT N
140  B0=P
149  REM - COMPUTE ALL PAYMENTS, BALANCES THROUGH LAST PAYMENT USING R
150  FOR J1=1  TO N*Y
159  REM - ROUND OFF INTEREST PAID TO NEAREST CENT
160  I1=INT((B0*I/N)*100+.5)/100
169  REM - CALCULATE AMOUNT AMORTIZED WITH EACH PAYMENT
170  A=R-I1
179  REM - BALANCE REMAINING DECREASES WITH EACH PAYMENT
180  B0=B0-A
190  NEXT J1
199  REM - CALCULATE LAST PAYMENT, ROUND OFF, PRINT
200  PRINT "LAST PAYMENT = $" ;INT((R+B0)*100+.5)/100
210  PRINT
219  REM - RESTART OR END PROGRAM?
220  PRINT "MORE DATA? (1=YES,0=NO)";
230  INPUT X
240  IF X=1  THEN   20
250  END
```

OPTION

The program above allows the term of payment on the loan to be entered in years only. You may wish to enter the term in years and months instead. The program changes necessary are listed following the example.

Example:

If you pay $40.00 a month for 2 years and 3 months on a loan of $1200.00 at 7.5%, what amount will the last payment total?

```
:RUN
LAST PAYMENT ON A LOAN

REGULAR PAYMENT? 40
PRINCIPAL? 1200
TERM IN YEARS AND MONTHS? 2,3
ANNUAL INTEREST RATE? 7.5
NUMBER OF PAYMENTS PER YEAR? 12
LAST PAYMENT = $ 287.36

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1   REM - OPTION 70-85
 10   PRINT  "LAST PAYMENT ON A LOAN"
  :
 60   INPUT P
 70   PRINT  "TERM IN YEARS AND MONTHS";
 80   INPUT Y0,M
 84   REM - CALCULATE YEARS FROM YEARS AND MONTHS
 85   Y=(12*Y0+M)/12
 90   PRINT  "ANNUAL INTEREST RATE";
  :
250   END
```

Remaining Balance on a Loan

This program calculates the balance remaining on a loan after a specified number of payments. It is necessary for you to provide the amount of the regular payment, the number of payments per year, the amount of the principal, the annual interest rate, and the payment number after which to calculate the remaining balance.

The remaining balance is calculated by the following method:

$$\text{remaining balance} = \text{principal} - \text{amount amortized after } N \cdot (Y - 1) + N1 \text{ payments}$$

where: N = number of payments per year
Y = year to calculate remaining balance
$N1$ = payment number in year Y to calculate remaining balance

Example:

Kelly has taken out a loan of $8000.00 at 17.2% interest. His regular payments are $200.00 per month. If he has paid through the tenth payment in the fourth year, how much more does Kelly owe on his loan?

```
:RUN
REMAINING BALANCE ON A LOAN

REGULAR PAYMENT? 200
PRINCIPAL? 8000
NUMBER OF PAYMENTS PER YEAR? 12
ANNUAL INTEREST RATE? 17.2
LAST PAYMENT MADE (PAYMENT NO.,YEAR)? 10,4
REMAINING BALANCE = $ 2496.17

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "REMAINING BALANCE ON A LOAN"
 20  PRINT
 29  REM - STATEMENTS 30 TO 130 REQUEST USER INPUT
 30  PRINT "REGULAR PAYMENT";
 40  INPUT R
 50  PRINT "PRINCIPAL";
 60  INPUT P
 70  PRINT "NUMBER OF PAYMENTS PER YEAR";
 80  INPUT N
 90  PRINT "ANNUAL INTEREST RATE";
100  INPUT I
109  REM - CONVERT FROM PERCENT TO DECIMAL
110  I=I/100
119  REM - ENTER THE PAYMENT NUMBER WITHIN THE YEAR, I.E. N1<=N
120  PRINT "LAST PAYMENT MADE (PAYMENT NO.,YEAR)";
130  INPUT N1,Y
139  REM - INITIALIZE REMAINING BALANCE
140  B0=P
149  REM - LOOP TO ACCUMULATE AMOUNT PAID SO FAR
```

```
150  FOR J1=1  TO N*(Y-1)+N1
159  REM - CALCULATE INTEREST PAID WITH EACH PAYMENT
160  I1=INT((B0*I/N)*100+.5)/100
169  REM - CALCULATE AMOUNT AMORTIZED WITH EACH PAYMENT
170  A=R-I1
179  REM - CALCULATE REMAINING BALANCE ON PRINCIPAL
180  B0=B0-A
190  NEXT J1
199  REM - ROUND OFF, PRINT
200  PRINT "REMAINING BALANCE = $";INT(B0*100+.5)/100
210  PRINT
219  REM - RESTART OR END PROGRAM?
220  PRINT "MORE DATA? (1=YES,0=NO)";
230  INPUT X
240  IF X=1  THEN   20
250  END
```

OPTION

You may wish to specify the number of the last payment made as the total payment number rather than the payment number within a certain year. For instance, when 4 payments are made per year, payment 3 of year 3 would be entered as payment number 11. The program changes necessary are listed following the example below.

Example:

John made ten quarterly payments of $550.00 on a loan of $6000.00 with 16% interest. What is his remaining balance?

```
:RUN
REMAINING BALANCE ON A LOAN

REGULAR PAYMENT? 550
PRINCIPAL? 6000
NUMBER OF PAYMENTS PER YEAR? 4
ANNUAL INTEREST RATE? 16
NUMBER OF PAYMENTS MADE? 10
REMAINING BALANCE = $ 2278.09

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

```
PROGRAM LISTING

  1  REM - OPTION 119-130, 150
 10  PRINT "REMAINING BALANCE ON A LOAN"
  :
110  I=I/100
119  REM - ENTER THE TOTAL NUMBER OF PAYMENTS MADE TO DATE
120  PRINT "NUMBER OF PAYMENTS MADE";
130  INPUT N1
139  REM - INITIALIZE REMAINING BALANCE
140  B0=P
149  REM - LOOP TO ACCUMULATE AMOUNT PAID SO FAR
150  FOR J1=1  TO N1
159  REM - CALCULATE INTEREST PAID WITH EACH PAYMENT
  :
250  END
```

Term of a Loan

This program calculates the period of time needed to repay a loan. You must specify the amount of the loan, the amount of the payments, the number of payments to be made per year and the annual interest rate on the loan. All payments are assumed to be equal.

The term of payment is derived from the following formula:

$$Y = -\frac{\log\left(1 - \frac{P \cdot i}{N \cdot R}\right)}{\log\left(1 + \frac{i}{N}\right)} \cdot \frac{1}{N}$$

where: Y = term of payment in years
P = principal
i = annual interest rate
N = number of payments per year
R = amount of payments

Examples:

What would be the duration of payment on a mortgage of $20,000.00 at 12% when payments of $1000.00 are to be made quarterly?

Sally takes out a loan for $12,669.00 at 16.8%. Her payments are $512.34 every two months. What is the term of her loan?

```
:RUN
TERM OF A LOAN

REGULAR PAYMENT? 1000
PRINCIPAL? 20000
ANNUAL INTEREST RATE? 18
NUMBER OF PAYMENTS PER YEAR? 4
TERM = 13.1 YEARS

MORE DATA? (1=YES, 0=NO)? 1

REGULAR PAYMENT? 512.34
PRINCIPAL? 12669
ANNUAL INTEREST RATE? 16.8
NUMBER OF PAYMENTS PER YEAR? 6
TERM = 7.1 YEARS

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "TERM OF A LOAN"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "REGULAR PAYMENT";
 40  INPUT R
 50  PRINT "PRINCIPAL";
 60  INPUT P
 70  PRINT "ANNUAL INTEREST RATE";
 80  INPUT I
 90  PRINT "NUMBER OF PAYMENTS PER YEAR";
100  INPUT N
108  REM - CALCULATE TERM IN YEARS BY FORMULA;
109  REM - INTEREST CONVERTED FROM PERCENT TO DECIMAL FOR CALCULATION
110  Y=-(LOG(1-(P*(I/100))/(N*R))/(LOG(1+I/100/N)*N))
119  REM - ROUND OFF TO NEAREST TENTH, PRINT
120  PRINT "TERM =";INT(Y*10+.5)/10;"YEARS"
130  PRINT
139  REM - RESTART OR END PROGRAM?
140  PRINT "MORE DATA? (1=YES, 0=NO)";
150  INPUT X
160  IF X=1  THEN   20
170  END
```

OPTION

It is possible to calculate the term of payment in years and months rather than just years. To do this, make the program changes listed following the example below.

Example:

Dick took out a loan for $8000.00 at 7.5%. Regular payments of $150.00 are to be made monthly. How long will it take to pay off the loan?

```
:RUN
TERM OF A LOAN

REGULAR PAYMENT? 150
PRINCIPAL? 8000
ANNUAL INTEREST RATE? 7.5
NUMBER OF PAYMENTS PER YEAR? 12
TERM = 5 YEARS, 5 MONTHS

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
   1  REM - OPTION 114-120
  10  PRINT "TERM OF A LOAN"
   :
 110  Y=-(LOG(1-(P*(I/100))/(N*R))/(LOG(1+I/100/N)*N))
 114  REM - CALCULATE YEARS AND MONTHS FROM YEARS
 115  M=INT(Y*12+.5)
 116  Y0=INT(M/12)
 117  M=M-Y0*12
 119  REM - PRINT RESULTS
 120  PRINT "TERM =";Y0;"YEARS,";M;"MONTHS"
 130  PRINT
   :
 170  END
```

Annual Interest Rate on a Loan

This program calculates the rate at which interest is charged on a loan. To determine this rate you must enter the amount of the loan, the amount of the regular payment, the number of payments per year, and the term of the loan.

The annual interest rate is computed by the following method of approximation:

1) Guess an interest rate
   Initialize last guess to 0
2) Compute regular payment using guessed rate:

$$R_1 = \frac{i \cdot P/N}{1-(1+i/N)^{-N \cdot Y}}$$

   Round off R_1
3) If computed payment = actual payment, then current guess = approximate interest rate
4) Otherwise, save current guess and calculate a new guess

$$i_2 = i$$

$$i = i \pm |(i-i_2)/2| \begin{cases} + \text{ if } R_1 < R \\ - \text{ if } R_1 > R \end{cases}$$

5) Go to 2

where:
- i = interest rate
- i_2 = previous interest rate
- R = input regular payment
- R_1 = computed regular payment
- P = principal
- N = number of payments per year
- Y = number of years

Examples:

Cindy borrowed $3000.00 from her friend George with an agreement to pay back $400.00 quarterly for 2 years. At what interest rate is she being charged?

To pay back a loan of $10,000.00 John contracted to make monthly payments of $120.00 for 9.5 years. At what rate is interest being charged?

```
:RUN
ANNUAL INTEREST RATE ON A LOAN

REGULAR PAYMENT? 400
TERM IN YEARS? 2
PRINCIPAL? 3000
NUMBER OF PAYMENTS PER YEAR? 4
ANNUAL INTEREST RATE = 5.827 %

MORE DATA? (1=YES, 0=NO)? 1

REGULAR PAYMENT? 120
TERM IN YEARS? 9.5
PRINCIPAL? 10000
NUMBER OF PAYMENTS PER YEAR? 12
ANNUAL INTEREST RATE = 6.933 %

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT  "AN   AL INTEREST RATE ON A LOAN"
 20  PRINT
 29  REM - STATEMENTS 30 TO 100 REQUEST USER INPUT
 30  PRINT "REGULAR PAYMENT";
 40  INPUT R
 50  PRINT "TERM IN YEARS";
 60  INPUT Y
 70  PRINT "PRINCIPAL";
 80  INPUT P
 90  PRINT  "NUMBER OF PAYMENTS PER YEAR";
100  INPUT N
109  REM - GUESS AN INTEREST RATE (10%) TO INITIATE TESTING
110  I=10
119  REM - I2=LAST GUESS OR ESTIMATE (START WITH 0)
120  I2=0
129  REM - COMPUTE REGULAR PAYMENT USING GUESSED INTEREST RATE
130  R1=(I*P/N)/(1-1/((I/N+1)↑(N*Y)))
139  REM - ROUND OFF TO NEAREST CENT
140  R1=INT(R1*100+.5)/100
149  REM - I3=NUMBER USED TO CLOSE IN ON INTEREST RATE
150  I3=ABS(I-I2)/2
159  REM - SAVE THIS GUESS
160  I2=I
168  REM - COMPARE COMPUTED PAYMENT (R1) TO INPUT PAYMENT (R);
169  REM - IF THEY'RE EQUAL, LAST RATE GUESSED=APPROXIMATE INT. RATE
170  IF R1=R  THEN  230
180  IF R1>R  THEN  210
189  REM - R1<R, RATE MUST BE HIGHER THAN LAST GUESS
190  I=I+I3
199  REM - RETEST WITH NEW GUESS
200  GOTO  130
209  REM - R1>R, RATE MUST BE LOWER THAN LAST GUESS
210  I=I-I3
219  REM - RETEST WITH NEW GUESS
220  GOTO  130
229  REM - COMPUTE INTEREST TO PROPER PROPORTIONS, ROUND OFF, PRINT
230  I=((INT((I*1000)*100+.5))/100)/1000
240  PRINT "ANNUAL INTEREST RATE =";I*100;"%"
250  PRINT
259  REM - RESTART OR END PROGRAM?
260  PRINT "MORE DATA? (1=YES, 0=NO)";
270  INPUT X
280  IF X=1  THEN   20
290  END
```

OPTION

The above listing allows the term of the loan to be entered in years only. You may wish to enter the term in years and months rather than years. The program changes necessary are listed following the example below.

Example:

If Connie pays $100.00 per month for 11 years and 7 months on a $10,000.00 loan, what is the annual interest rate on the loan?

```
:RUN
ANNUAL INTEREST RATE ON A LOAN

REGULAR PAYMENT? 100
TERM IN YEARS, MONTHS? 11,7
PRINCIPAL? 10000
NUMBER OF PAYMENTS PER YEAR? 12
ANNUAL INTEREST RATE = 6.002 %

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 50-65
 10  PRINT  "ANNUAL INTEREST RATE ON A LOAN"
  :
 40  INPUT R
 50  PRINT "TERM IN YEARS, MONTHS";
 60  INPUT Y0,M
 64  REM - CALCULATE YEARS FROM YEARS AND MONTHS
 65  Y=(12*Y0+M)/12
 70  PRINT "PRINCIPAL";
  :
290  END
```

Mortgage Amortization Table

This program calculates and prints a loan repayment schedule. This schedule provides the following outputs:

1) Payment number
2) Amount of each payment paid as interest
3) Amount of the loan amortized with each payment
4) Balance remaining on the principal at the time of each payment
5) Accumulated interest paid at the time of each payment
6) Amount of the last payment

In addition, the yearly totals of interest paid and amount amortized are tabulated and printed.

To use this program you must supply the amount of the regular payrnent, the term of payment, the number of payments per year, the amount of the principal and the annual interest rate.

The schedule is calculated in the following manner:

1) Payment number = payment number within each year
2) Amount of each payment paid as interest = remaining balance $\cdot\ i/N$
   where: i = annual interest rate
   N = number of payments per year
3) Amount amortized with each payment $= R - I$
   where: R = amount of regular payment
   I = amount of each payment paid as interest
4) Balance remaining $= P - \Sigma A$
   where: P = principal
   ΣA = sum of amounts amortized with each payment to date
5) Accumulated interest $= \Sigma I$
   where: ΣI = sum of amounts of each payment paid as interest to date
6) Amount of last payment $= R + (P - R \cdot N \cdot Y)$
   where: R = regular payment
   P = principal
   N = number of payments per year
   Y = number of years

Example:

David needs $2100.00 to pay off some debts. His sister offers him the money at 6% interest. With payments of $75.00 monthly for 2½ years, what is David's repayment schedule?

```
:RUN
MORTGAGE AMORTIZATION TABLE

REGULAR PAYMENT? 75
TERM IN YEARS? 2.5
PRINCIPAL? 2100
ANNUAL INTEREST RATE? 6
NUMBER OF PAYMENTS PER YEAR? 12
START PRINTING WITH WHAT YEAR? 1
```

```
                    MORTGAGE AMORTIZATION TABLE
      PRINCIPAL $ 2100      AT 6 %     FOR 2.5 YEARS
                    REGULAR PAYMENT = $ 75

NO.     INTEREST   AMORTIZED        BALANCE          ACCUM INTEREST
 1       10.5       64.5            2035.5            10.5
 2       10.18      64.82           1970.68           20.68
 3       9.85       65.15           1905.53           30.53
 4       9.53       65.47           1840.06           40.06
 5       9.2        65.8            1774.26           49.26
 6       8.87       66.13           1708.13           58.13
 7       8.54       66.46           1641.67           66.67
 8       8.21       66.79           1574.88           74.88
 9       7.87       67.13           1507.75           82.75
 10       7.54      67.46           1440.29           90.29
 11       7.2       67.8            1372.49           97.49
 12       6.86      68.14           1304.35           104.35

YR. 1   104.35     795.65

 1       6.52       68.48           1235.87           110.87
 2       6.18       68.82           1167.05           117.05
 3       5.84       69.16           1097.89           122.89
 4       5.49       69.51           1028.38           128.38
 5       5.14       69.86           958.52            133.52
 6       4.79       70.21           888.31            138.31
 7       4.44       70.56           817.75            142.75
 8       4.09       70.91           746.84            146.84
 9       3.73       71.27           675.57            150.57
 10       3.38      71.62           603.95            153.95
 11       3.02      71.98           531.97            156.97
 12       2.66      72.34           459.63            159.63

YR. 2   55.28      844.72

 1       2.3        72.7            386.93            161.93
 2       1.93       73.07           313.86            163.86
 3       1.57       73.43           240.43            165.43
 4       1.2        73.8            166.63            166.63
 5       .83        74.17           92.46             167.46
 6       .46        92.46           0                 167.92
         LAST PAYMENT = $ 92.92

YR. 3   8.29       459.63

CHANGE DATA AND RECOMPUTE? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  10  PRINT "MORTGAGE AMORTIZATION TABLE"
  20  PRINT
  29  REM - STATEMENTS 30 TO 150 REQUEST USER INPUT
  30  PRINT "REGULAR PAYMENT";
  40  INPUT R
```

```
 50  PRINT "TERM IN YEARS";
 60  INPUT Y
 70  PRINT "PRINCIPAL";
 80  INPUT P
 90  PRINT "ANNUAL INTEREST RATE";
100  INPUT I
109  REM - CONVERT FROM PERCENT TO DECIMAL
110  I=I/100
120  PRINT "NUMBER OF PAYMENTS PER YEAR";
130  INPUT N
140  PRINT  "START PRINTING WITH WHAT YEAR";
150  INPUT X
159  REM - START PRINTING AT BEGINNING OF A YEAR
160  X=INT(X)
169  REM - INITIALIZE VARIABLES
170  C1=0
180  I2=0
190  I3=0
200  J0=0
210  N1=N
220  K=66
230  B0=P
240  A1=0
250  A2=0
259  REM - TERM LESS THAN ONE YEAR?
260  IF INT(Y)>=1  THEN  270
261  REM - ADJUST VARIABLES TO PRINT A PARTIAL YEAR
262  N1=((Y-INT(Y))*12)/12*N
263  J0=J0+1
264  GOTO  280
269  REM - LOOP FOR EACH YEAR
270  FOR J0=1  TO INT(Y)
279  REM - START PRINTING?
280  IF J0<X  THEN  410
289  REM - NEED TO START NEXT PAGE?
290  IF K+N+3<58  THEN  400
299  REM - SPACE TO TOP OF NEXT PAGE (ASSUME 66 LINES PER PAGE)
300  FOR K1=K  TO 66
310  PRINT
320  NEXT K1
330  PRINT
339  REM - PRINT PAGE HEADINGS
340  PRINT  "                              MORTGAGE AMORTIZATION TABLE"
350  PRINT "     PRINCIPAL $";P;"     AT";I*100;"%     FOR";Y; "YEARS"
360  PRINT  "                              REGULAR PAYMENT = $";R
370  PRINT
380  PRINT  "NO.    ";"INTEREST","AMORTIZED","BALANCE","ACCUM INTEREST"
389  REM - COUNT LINES PRINTED ON EACH PAGE IN K
390  K=7
400  K=K+N+3
410  FOR J1=1  TO N1
419  REM - CALCULATE INTEREST PAID THIS PAYMENT, ROUND OFF
420  I1=INT((B0*I/N)*100+.5)/100
429  REM - COUNT NUMBER OF PAYMENTS MADE SO FAR
430  C1=C1+1
439  REM - CALCULATE AMOUNT AMORTIZED THIS PAYMENT
440  A=R-I1
```

```
449  REM - SUM AMOUNT AMORTIZED TO DATE
450  A1=A1+A
459  REM - CALCULATE BALANCE DUE
460  B0=P-A1
468  REM - LAST PAYMENT? IF YES, CALCULATE AMOUNT SO THAT THE
469  REM - BALANCE DUE EQUALS $00.00 AFTER THIS PAYMENT
470  IF C1<>N*Y  THEN  520
480  R=R+B0
490  A=A+B0
500  A1=A1+B0
510  B0=0
519  REM - SUM INTEREST PAID TO DATE
520  I2=I2+I1
529  REM - SUM INTEREST PAID THIS YEAR
530  I3=I3+I1
539  REM - SUM AMOUNT AMORTIZED THIS YEAR
540  A2=A2+A
549  REM - STARTED PRINTING? IF YES, PRINT COMPUTED VALUES IN TABLE
550  IF J0<X  THEN  570
560  PRINT J1;"    ";I1,A,B0,I2
570  NEXT J1
579  REM - LAST PAYMENT? IF YES, ROUND OFF, PRINT
580  IF C1<>N*Y  THEN  600
590  PRINT  "        LAST PAYMENT = $";(INT(R*100+.5))/100
599  REM - STARTED PRINTING? IF YES, PRINT YEARLY TOTALS
600  IF J0<X  THEN  640
610  PRINT
620  PRINT "YR.";J0;I3,A2
630  PRINT
639  REM - COMPLETED TERM?
640  IF J0>Y  THEN  720
649  REM - REINITIALIZE YEARLY VARIABLES
650  I3=0
660  A2=0
670  NEXT J0
679  REM - NEED TO PRINT A PARTIAL YEAR?
680  IF Y<>J0  THEN  262
720  PRINT
729  REM - RESTART OR END PROGRAM?
730  PRINT "CHANGE DATA AND RECOMPUTE? (1=YES, 0=NO)";
740  INPUT Z
750  IF Z=1  THEN  20
760  END
```

OPTION

You may wish to enter the term of payment in years and months rather than years. The program changes necessary are listed following the example below.

Example:

If you took out a loan for $700.00 from a friend at 9% interest and were to pay $100.00 per month for 8 months, what would your repayment schedule be?

```
:RUN
MORTGAGE AMORTIZATION TABLE

REGULAR PAYMENT? 100
TERM IN YEARS, MONTHS? 0,8
PRINCIPAL? 700
ANNUAL INTEREST RATE? 9
NUMBER OF PAYMENTS PER YEAR? 12
START PRINTING WITH WHAT YEAR? 1


                        MORTGAGE AMORTIZATION TABLE
 PRINCIPAL $ 700        AT 9 %    FOR 0 YEARS 8 MONTHS
                        REGULAR PAYMENT = $ 100

NO.    INTEREST  AMORTIZED        BALANCE          ACCUM INTEREST
 1      5.25       94.75           605.25           5.25
 2      4.54       95.46           509.79           9.79
 3      3.82       96.18           413.61           13.61
 4      3.1        96.9            316.71           16.71
 5      2.38       97.62           219.09           19.09
 6      1.64       98.36           120.73           20.73
 7      .91        99.09           21.64            21.64
 8      .16        21.64           0                21.8
        LAST PAYMENT = $ 21.8

YR. 1  21.8        700


CHANGE DATA AND RECOMPUTE? (1=YES, 0=NO)? 0

END PROGRAM


PROGRAM LISTING

   1  REM - OPTION 50-65,350
  10  PRINT "MORTGAGE AMORTIZATION TABLE"
   :
  40  INPUT R
  50  PRINT "TERM IN YEARS, MONTHS";
  60  INPUT Y0,M
  64  REM - CONVERT YEARS AND MONTHS TO YEARS
  65  Y=(12*Y0+M)/12
  70  PRINT "PRINCIPAL";
   :
 340  PRINT "                          MORTGAGE AMORTIZATION TABLE"
 350  PRINT " PRINCIPAL $";P;"    AT";I*100;"%    FOR";Y0;"YEARS";M;"MO
      NTHS"
 360  PRINT "                          REGULAR PAYMENT = $";R
   :
 760  END
```

Greatest Common Denominator

This program calculates the greatest common denominator of two integers. It is based on the Euclidean algorithm for finding the GCD:

1) Enter A, B
   A = absolute value of A
   B = absolute value of B
2) Calculate $R = A - B \cdot$ (integer of (A/B))
3) Is $R = 0$?. If yes, the GCD = B
   If no, go to step 4
4) $A = B$
   $B = R$
5) Go to step 2

Example:

Find the greatest common denominator of 50 and 18, 115 and 150.

```
:RUN
GREATEST COMMON DENOMINATOR

(ENTER 0,0 TO END PROGRAM)
ENTER TWO NUMBERS? 50,18
G.C.D: 2

ENTER TWO NUMBERS? 115,150
G.C.D: 5

ENTER TWO NUMBERS? 0,0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "GREATEST COMMON DENOMINATOR"
 20  PRINT
 30  PRINT "(ENTER 0,0 TO END PROGRAM)"
 40  PRINT "ENTER TWO NUMBERS";
 50  INPUT A,B
 59  REM - END PROGRAM?
 60  IF A<>0  THEN   90
 70  IF B<>0  THEN   90
 80  GOTO  150
 89  REM - CALCULATE GCD ACCORDING TO EUCLIDEAN ALGORITHM, PRINT RESULT
 90  A=ABS(A)
100  B=ABS(B)
110  R=A-B*INT(A/B)
120  IF R=0  THEN  160
130  A=B
140  B=R
```

```
150  GOTO  110
160  PRINT "G.C.D:";B
169  REM - PRINT BLANK LINE TO SEPARATE SETS OF DATA
170  PRINT
179  REM - RESTART PROGRAM
180  GOTO   40
190  END
```

Prime Factors of Integers

This program lists the prime factors of an integer. It will not test for the integer 0.

Examples:

What are the prime factors of -49?

Factor 92 into primes.

```
RUN
PRIME FACTORS OF INTEGERS
(ENTER 0 TO END PROGRAM)
NUMBER? -49
 1
 7 ↑ 2
NUMBER? 92
 1
 2 ↑ 2
 23 ↑ 1
NUMBER? 0
END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "PRIME FACTORS OF INTEGERS"
 20  PRINT
 30  PRINT "(ENTER 0 TO END PROGRAM)"
 40  PRINT "NUMBER";
 50  INPUT Z
 59  REM - END PROGRAM?
 60  IF Z=0  THEN  200
 69  REM - THE SIGN OF THE NUMBER IS ALWAYS A FACTOR
 70  PRINT SGN(Z)
 79  REM - USE ABSOLUTE VALUE FOR CALCULATIONS
 80  Z=ABS(Z)
 88  REM - LOOP TO TEST ALL INTEGERS (2 THROUGH Z) AS PRIME FACTORS
 89  REM - INTEGERS SQR(Z) THRU Z WILL HAVE NO NEW FACTORS
 90  FOR I=2  TO SQR(Z)
100  S=0
110  IF Z/I<>INT(Z/I)  THEN  150
120  Z=Z/I
130  S=S+1
140  GOTO  110
149  REM - FIND A PRIME FACTOR? IF YES, PRINT
150  IF S=0  THEN  170
159  REM - PRINT FACTORS WITH EXPONENTS; I↑S = I TO THE S POWER
160  PRINT I;"↑";S
170  NEXT I
180  PRINT
189  REM - RESTART PROGRAM
190  GOTO  40
200  END
```

Area of a Polygon

This program calculates the area of a polygon. You must supply the x - and y -coordinates of all vertices. Coordinates must be entered in order of successive vertices.

The formula used to calculate the area is:

$$\text{Area} = (x_1 + x_2)\cdot(y_1 - y_2) + (x_2 + x_3)\cdot(y_2 - y_3) + \ldots (x_n + x_1)\cdot(y_n - y_1)\,12$$

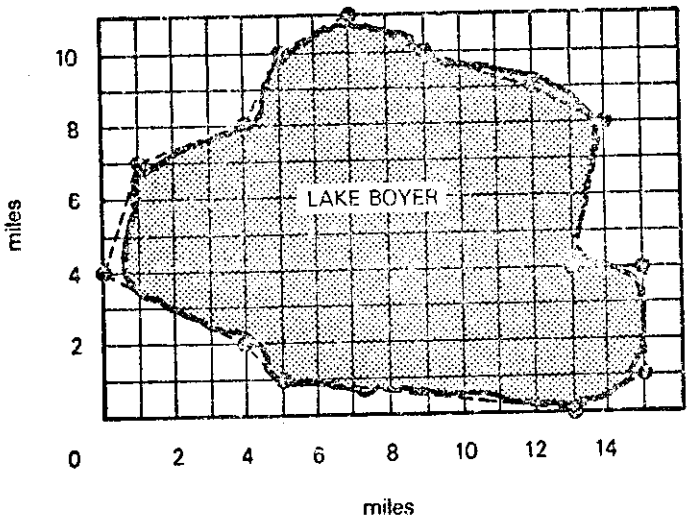
where n = the number of vertices.

The number of vertices you may enter is currently limited to 24. You may increase or decrease this limit by altering statement 30 according to the following scheme:

```
30 DIM X(n+1), Y(n+1)
```

Example:

Approximate the area of Lake Boyer.




```
:RUN
AREA OF A POLYGON

NUMBER OF VERTICES (ENTER 0 TO END PROGRAM)? 14
COORDINATES OF VERTEX 1 ? 0,4
                  VERTEX 2 ? 1,7
                  VERTEX 3 ? 4,8
                  VERTEX 4 ? 5,10
                  VERTEX 5 ? 7,11
                  VERTEX 6 ? 9,10
                  VERTEX 7 ? 12,9
```

```
                VERTEX 8 ? 14,8
                VERTEX 9 ? 13,4
                VERTEX 10 ? 15,4
                VERTEX 11 ? 15,1
                VERTEX 12 ? 13,0
                VERTEX 13 ? 5,1
                VERTEX 14 ? 4,2
AREA = 108

NUMBER OF VERTICES (ENTER 0 TO END PROGRAM)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "AREA OF A POLYGON"
 20  PRINT
 29  REM - COORDINATE ARRAYS SHOULD BE SET TO (NUMBER OF VERTICES +1)
 30  DIM X(25),Y(25)
 40  PRINT "NUMBER OF VERTICES (ENTER 0 TO END PROGRAM)";
 50  INPUT N
 59  REM - END PROGRAM?
 60  IF N=0  THEN  230
 69  REM - LOOP TO ENTER COORDINATES IN ORDER OF SUCCESSIVE VERTICES
 70  FOR I=1  TO N
 80  IF I>1  THEN  110
 90  PRINT "COORDINATES OF VERTEX";I;
100  GOTO  120
110  PRINT "                VERTEX";I;
120  INPUT X(I),Y(I)
130  NEXT I
139  REM - FIRST VERTEX SERVES AS LAST VERTEX
140  X(N+1)=X(1)
150  Y(N+1)=Y(1)
160  A=0
169  REM - CALCULATE AREA, PRINT
170  FOR I=1  TO N
180  A=A+(X(I)+X(I+1))*(Y(I)-Y(I+1))
190  NEXT I
200  PRINT "AREA =";ABS(A)/2
210  PRINT
219  REM - RESTART PROGRAM
220  GOTO   40
230  END
```

Parts of a Triangle

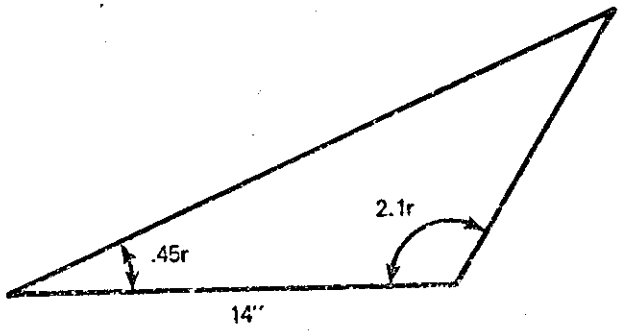
This program calculates three unknown parts of a triangle when three parts are given. At least one part given must be the length of a side. There are five possibilities for data entry:

1) Angle, side, angle
2) Side, angle, side
3) Angle, angle, side
4) Side, side, angle
5) Side, side, side

Data must be entered in the order it appears in a triangle, either clockwise or counterclockwise.

Example:

The base of a triangle measures 14 inches. The base angles measure .45 and 2.1 radians. What are the measurements of the triangle?




```
:RUN
PARTS OF A TRIANGLE

PROBLEM TYPES: 1=ASA,2=SAS,3=AAS,4=SSA,5=SSS,6=END PROGRAM
ENTER PROBLEM TYPE? 1
ENTER ANGLE,SIDE,ANGLE? .45,14,2.1

SIDE 1 = 10.919
OPPOSITE ANGLE= .45 RADIANS
SIDE 2 = 21.67
OPPOSITE ANGLE= 2.1 RADIANS
SIDE 3 = 14
OPPOSITE ANGLE= .592 RADIANS

ENTER PROBLEM TYPE? 6

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "PARTS OF A TRIANGLE"
 20  PRINT
 30  DIM A(3),S(3)
 31  REM - SET VALUE OF PI
 40  P=3.1415927
 48  REM - ENTER NUMBER OF PROBLEM TYPE ACCORDING TO KNOWN PARTS
 49  REM - OF THE TRIANGLE WHERE A=ANGLE, S=LENGTH OF SIDE
 50  PRINT "PROBLEM TYPES: 1=ASA,2=SAS,3=AAS,4=SSA,5=SSS,6=END PROGRAM
     "
 60  PRINT "ENTER PROBLEM TYPE";
 70  INPUT X
 79  REM - DIRECT PROGRAM TO PROPER CALCULATIONS
 80  IF X=6  THEN  560
 90  IF X=5  THEN  390
100  IF X=4  THEN  300
110  IF X=3  THEN  260
120  IF X=2  THEN  190
130  PRINT "ENTER ANGLE,SIDE,ANGLE";
140  INPUT A(1),S(3),A(2)
150  A(3)=P-A(1)-A(2)
160  S(1)=S(3)*SIN(A(1))/SIN(A(3))
170  S(2)=S(3)*SIN(A(2))/SIN(A(3))
180  GOTO  440
190  PRINT "ENTER SIDE,ANGLE,SIDE";
200  INPUT S(3),A(1),S(2)
210  S(1)=SQR(S(3)↑2+S(2)↑2-2*S(3)*S(2)*COS(A(1)))
220  A(2)=SIN(A(1))/S(1)*S(2)
230  A(2)=ARCSIN(A(2))
240  A(3)=P-A(1)-A(2)
250  GOTO  440
260  PRINT "ENTER ANGLE,ANGLE,SIDE";
270  INPUT A(3),A(2),S(3)
280  A(1)=P-A(2)-A(3)
290  GOTO  160
300  PRINT "ENTER SIDE,SIDE,ANGLE";
310  INPUT S(1),S(2),A(1)
320  T=S(2)*SIN(A(1))
330  IF S(1)<T  THEN  520
340  S(3)=SQR(S(2)↑2-T↑2)
350  IF S(1)<=T  THEN  380
360  Y=SQR(S(1)↑2-T↑2)
370  S(3)=S(3)+Y
380  GOTO  220
390  PRINT "ENTER SIDE,SIDE,SIDE";
400  INPUT S(1),S(2),S(3)
410  A(1)=(S(2)↑2+S(3)↑2-S(1)↑2)/2/S(2)/S(3)
420  A(1)=ARCCOS(A(1))
430  GOTO  220
440  PRINT
449  REM - PRINT RESULTS
450  FOR I=1  TO 3
459  REM - THE ANGLE OF A TRIANGLE CANNOT BE LESS THAN ZERO
460  IF A(I)<0  THEN  520
470  PRINT "SIDE";I;"=";INT(S(I)*1000+.5)/1000
480  PRINT "OPPOSITE ANGLE=";INT(A(I)*1000+.5)/1000;"RADIANS"
```

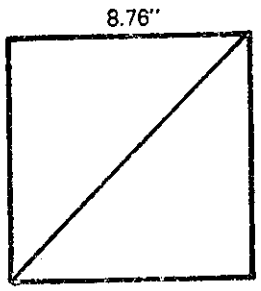
```
490   NEXT I
500   PRINT
510   GOTO   60
520   PRINT
530   PRINT "NO SOLUTION"
540   PRINT
550   GOTO   60
560   END
```

OPTION

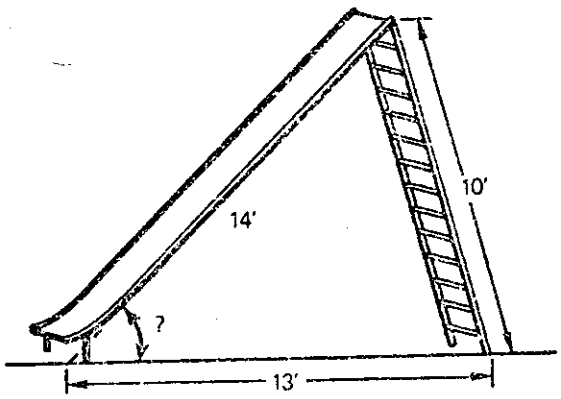
It may be more convenient for you to work with angles in degrees rather than radians. The program changes necessary are listed following the examples below.

Examples:

A square measures 8.76" x 8.76". What is the length of its diagonal?



The ladder of a slide measures 10', the slide 14', and it covers 13' of ground from base of ladder to tip o slide. How steep is the slide?




```
:RUN
PARTS OF A TRIANGLE

PROBLEM TYPES: 1=ASA,2=SAS,3=AAS,4=SSA,5=SSS,6=END PROGRAM
ENTER PROBLEM TYPE? 2
ENTER SIDE,ANGLE,SIDE? 8.76,90,8.76

SIDE 1 = 12.389
OPPOSITE ANGLE= 90 DEGREES
SIDE 2 = 8.76
OPPOSITE ANGLE= 45 DEGREES
SIDE 3 = 8.76
OPPOSITE ANGLE= 45 DEGREES
```

```
ENTER PROBLEM TYPE? 5
ENTER SIDE,SIDE,SIDE? 10,13,14

SIDE 1 = 10
OPPOSITE ANGLE= 43.279 DEGREES
SIDE 2 = 13
OPPOSITE ANGLE= 63.027 DEGREES
SIDE 3 = 14
OPPOSITE ANGLE= 73.694 DEGREES

ENTER PROBLEM TYPE? 6

END PROGRAM
```

PROGRAM LISTING

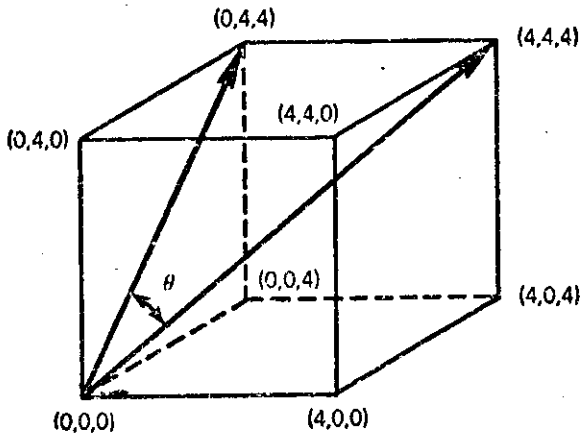
```
  1  REM - OPTION 44-45,145-146,205,275-276,305,480
 10  PRINT "PARTS OF A TRIANGLE"
  :
 40  P=3.1415927
 44  REM - SET CONVERSION FACTOR FOR CONVERTING DEGREES TO RADIANS
 45  C=.0174532927
 48  REM - ENTER NUMBER OF PROBLEM TYPE ACCORDING TO KNOWN PARTS
  :
140  INPUT A(1),S(3),A(2)
145  A(1)=A(1)*C
146  A(2)=A(2)*C
150  A(3)=P-A(1)-A(2)
  :
200  INPUT S(3),A(1),S(2)
205  A(1)=A(1)*C
210  S(1)=SQR(S(3)^2+S(2)^2-2*S(3)*S(2)*COS(A(1)))
  :
270  INPUT A(3),A(2),S(3)
275  A(3)=A(3)*C
276  A(2)=A(2)*C
280  A(1)=P-A(2)-A(3)
  :
310  INPUT S(1),S(2),A(1)
315  A(1)=A(1)*C
320  T=S(2)*SIN(A(1))
  :
470  PRINT "SIDE";I;"=";INT(S(I)*1000+.5)/1000
480  PRINT "OPPOSITE ANGLE=";INT(A(I)/C*1000+.5)/1000;"DEGREES"
490  NEXT I
  :
560  END
```

Analysis of Two Vectors

This program calculates the angle between two given vectors, the angle between each vector and axis, and the magnitude of each vector. The vectors are given in three dimensional space.

Example:

Find the angle (θ) between a diagonal of a cube and a diagonal of one of its faces. The cube measures 4 x 4 x 4.




```
:RUN
ANALYSIS OF TWO VECTORS

VECTOR 1: X,Y,Z? 0,4,4
VECTOR 2: X,Y,Z? 4,4,4

VECTOR 1 :
MAGNITUDE: 5.6568542495
ANGLE WITH X-AXIS: 90.00000076485
ANGLE WITH Y-AXIS: 45.00000033257
ANGLE WITH Z-AXIS: 45.00000038257

VECTOR 2 :
MAGNITUDE: 6.9282032303
ANGLE WITH X-AXIS: 54.73561073261
ANGLE WITH Y-AXIS: 54.73561078261
ANGLE WITH Z-AXIS: 54.73561073261

ANGLE BETWEEN VECTORS: 35.26438998282

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "ANALYSIS OF TWO VECTORS"
 20  PRINT
 30  DIM X(2), Y(2), Z(2), M(2)
 39  REM - STATEMENTS 40 TO 70 REQUEST VECTOR COORDINATES
 40  PRINT "VECTOR 1: X,Y,Z";
 50  INPUT X(1),Y(1),Z(1)
 60  PRINT "VECTOR 2: X,Y,Z";
 70  INPUT X(2),Y(2),Z(2)
 80  PRINT
 89  REM - LOOP TO ANALYZE BOTH VECTORS
 90  FOR I=1  TO 2
 99  REM - CALCULATE MAGNITUDE, PRINT
100  M(I)=SQR(X(I)↑2+Y(I)↑2+Z(I)↑2)
109  REM - IS VECTOR A POINT? IF YES, CANNOT COMPUTE AN ANGLE
110  IF M(I)=0  THEN  220
120  PRINT "VECTOR";I;":"
130  PRINT "MAGNITUDE:";M(I)
139  REM - CONVERSION FACTOR FOR RADIANS TO DEGREES
140  S=57.29578
149  REM - CALCULATE ANGLE BETWEEN VECTOR AND X-AXIS, PRINT
150  J=X(I)/M(I)
160  PRINT "ANGLE WITH X-AXIS:";ARCCOS(J)*S
169  REM - CALCULATE ANGLE BETWEEN VECTOR AND Y-AXIS, PRINT
170  J=Y(I)/M(I)
180  PRINT "ANGLE WITH Y-AXIS:";ARCCOS(J)*S
189  REM - CALCULATE ANGLE BETWEEN VECTOR AND Z-AXIS, PRINT
190  J=Z(I)/M(I)
200  PRINT "ANGLE WITH Z-AXIS:";ARCCOS(J)*S
210  PRINT
220  NEXT I
230  J=0
239  REM - IF EITHER VECTOR A POINT, CANNOT COMPUTE ANGLE
240  IF M(1)=0  THEN  310
250  IF M(2)=0  THEN  310
259  REM - CALCULATE ANGLE BETWEEN VECTORS
260  J=(X(1)*X(2)+Y(1)*Y(2)+Z(1)*Z(2))/M(1)/M(2)
269  REM - ARE THE VECTORS PERPENDICULAR?
270  IF J<>0  THEN  300
280  J=90
290  GOTO  310
299  REM - CALCULATE ANGLE IN DEGREES, PRINT
300  J=ARCCOS(J)*S
310  PRINT "ANGLE BETWEEN VECTORS:";J
320  PRINT
329  REM - RESTART OR END PROGRAM?
330  PRINT "MORE DATA (1=YES, 0=NO)";
340  INPUT Z
350  IF Z=1  THEN   20
360  END
```

Operations on Two Vectors

This program performs four operations on two vectors given in three space. The operations performed are

1) Addition
2) Subtraction
3) Scalar (dot) product
4) Cross product

Example:

Vectors are drawn from the origin to two points A(5,-1,2) and B(1,4,9). Add, subtract, and find the dot and cross product of these vectors.

```
:RUN
OPERATIONS ON TWO VECTORS

VECTOR A: X,Y,Z COORDINATES? 5,-1,2
VECTOR B: X,Y,Z COORDINATES? 1,4,9

A+B= 6 , 3 , 11
A-B= 4 ,-5 ,-7
A.B= 19
A*B=-17 ,-43 , 21

MORE DATA? (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "OPERATIONS ON TWO VECTORS"
 20  PRINT
 30  PRINT "VECTOR A: X,Y,Z COORDINATES";
 40  INPUT X1,Y1,Z1
 50  PRINT "VECTOR B: X,Y,Z COORDINATES";
 60  INPUT X2,Y2,Z2
 70  PRINT
 79  REM - PERFORM VECTOR ADDITION, PRINT RESULTING VECTOR COORDINATE
 80  PRINT "A+B=";X1+X2;",";Y1+Y2;",";Z1+Z2
 89  REM - PERFORM VECTOR SUBTRACTION, PRINT RESULTING VECTOR COORDINAT
 90  PRINT "A-B=";X1-X2;",";Y1-Y2;",";Z1-Z2
 99  REM - CALCULATE DOT PRODUCT, PRINT
100  PRINT "A.B=";X1*X2+Y1*Y2+Z1*Z2
109  REM - CALCULATE CROSS PRODUCT, PRINT RESULTING VECTOR COORDINATE
110  PRINT "A*B=";Y1*Z2-Z1*Y2;",";Z1*X2-X1*Z2;",";X1*Y2-Y1*X2
120  PRINT
129  REM - RESTART OR END PROGRAM?
130  PRINT "MORE DATA? (1=YES, 0=NO)";.
140  INPUT X
150  IF X=1  THEN   20
160  END
```

Angle Conversion: Radians to Degrees

This program converts an angle given in radians to degrees, minutes and seconds.

Example:

How many degrees, minutes and seconds are there in an angle of 2.5 radians? In 118 radians?

```
:RUN
ANGLE CONVERSION: RADIANS TO DEGREES

ANGLE IN RADIANS (ENTER 0 TO END PROGRAM)? 2.5
   DEGREES = 143
   MINUTES = 14
   SECONDS = 22.01

ANGLE IN RADIANS? 118
   DEGREES = 280
   MINUTES = 54
   SECONDS = 6.78

ANGLE IN RADIANS? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "ANGLE CONVERSION: RADIANS TO DEGREES"
 20  PRINT
 30  PRINT "ANGLE IN RADIANS (ENTER 0 TO END PROGRAM)";
 40  GOTO   60
 50  PRINT "ANGLE IN RADIANS";
 60  INPUT R
 69  REM - TEST FOR END OF PROGRAM
 70  IF R=0  THEN  170
 79  REM - CONVERT RADIANS TO SECONDS
 80  A=3600*180*R/3.1415927
 89  REM - CALCULATE NUMBER OF WHOLE DEGREES
 90  D=INT(A/3600)
 99  REM - CALCULATE NUMBER OF FULL CIRCLES
100  D1=INT(D/360)
109  REM - CALCULATE DEGREES OF ANGLE WITHIN 360 DEGREES, PRINT
110  PRINT "   DEGREES =";D-360*D1
119  REM - CALCULATE MINUTES, PRINT
120  PRINT "   MINUTES =";INT((A-D*3600)/60)
129  REM - CALCULATE SECONDS, ROUND OFF, PRINT
130  S=A-D*3600-(INT((A-D*3600)/60))*60
140  PRINT "   SECONDS =";INT(100*S+.5)/100
150  PRINT
159  REM RESTART PROGRAM
160  GOTO   50
170  END
```

OPTION

You may prefer your answer in degrees and decimals of degrees rather than degrees, minutes and seconds. The program changes necessary are listed following the example below.

Example:

How many degrees are there in an angle of 2.5 radians?

```
:RUN
ANGLE CONVERSION: RADIANS TO DEGREES

ANGLE IN RADIANS (ENTER 0 TO END PROGRAM)? 2.5
   DEGREES = 143

ANGLE IN RADIANS? 0

END PROGRAM
```

PROGRAM LISTING

```
   1  REM - OPTION 110
  10  PRINT "ANGLE CONVERSION: RADIANS TO DEGREES"
   :
 109  REM - CALCULATE DEGREES OF ANGLE WITHIN 360 DEGREES, PRINT
 110  PRINT "   DEGREES =";INT((D-360*D1)*100+.5)/100
 150  PRINT
   :
 170  END
```

Angle Conversion: Degrees to Radians

This program converts an angle given in degrees, minutes and seconds to radians.

Examples:

An angle measures 30 degrees, 5 minutes and 3 seconds. What would be the measure of this angle in radians?

What would be the radian measurement of two angles measuring 278°, 19', 54" and 721°, 0', 0"?

```
:RUN
ANGLE CONVERSION: DEGREES TO RADIANS

(TO END PROGRAM ENTER 0,0,0)
ANGLE IN DEGREES,MINUTES,SECONDS? 30,5,3
RADIANS =  .5250676852416

ANGLE IN DEGREES,MINUTES,SECONDS? 278,19,54
RADIANS =  4.857803294516

ANGLE IN DEGREES,MINUTES,SECONDS? 721,0,0
RADIANS =  1.74514900E-02

ANGLE IN DEGREES,MINUTES,SECONDS? 0,0,0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "ANGLE CONVERSION: DEGREES TO RADIANS"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER 0,0,0)"
 40  PRINT "ANGLE IN DEGREES,MINUTES,SECONDS";
 50  INPUT D,M,S
 59  REM - TEST FOR END OF PROGRAM
 60  IF D<>0  THEN  100
 70  IF M<>0  THEN  100
 80  IF S<>0  THEN  100
 90  GOTO  150
 99  REM - CONVERT DEGREES, MINUTES, SECONDS TO DEGREES
100  A=D+M/60+S/3600
109  REM - CALCULATE NUMBER OF COMPLETE CIRCLES
110  R=INT(A/360)
119  REM - CALCULATE ANGLE WITHIN 360 DEGREES, PRINT
120  PRINT "RADIANS =";A*.01745329-R*6.2831853
130  PRINT
139  REM - RESTART PROGRAM
140  GOTO   40
150  END
```

OPTION

It may be more convenient for you to enter the angle in degrees and fractions of degrees rather th degrees, minutes and seconds. The program changes necessary are listed following the example below.

Example:

How many radians are in an angle measuring 33.08°? 90°?

```
:RUN
ANGLE CONVERSION: DEGREES TO RADIANS

(TO END PROGRAM ENTER 0)
ANGLE IN DEGREES? 33.08
RADIANS = .5773548332

ANGLE IN DEGREES? 90
RADIANS = 1.5707961

ANGLE IN DEGREES? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 30-60
 10  PRINT "ANGLE CONVERSION: DEGREES TO RADIANS"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER 0)"
 40  PRINT "ANGLE IN DEGREES";
 50  INPUT A
 59  REM - TEST FOR END OF PROGRAM
 60  IF A=0  THEN  150
109  REM - CALCULATE NUMBER OF COMPLETE CIRCLES
  :
150  END
```

Coordinate Conversion

This program converts the coordinates of a point given in Cartesian coordinates to polar coordinates, and vice versa.

The formulas for the conversions are:

$$r = \sqrt{x^2 + y^2}$$

$$A = \text{arctangent}\,(y/x)$$

$$x = r \cdot \text{cosine}\,(A)$$

$$y = r \cdot \text{sine}\,(A)$$

where: x = abscissa, y = ordinate } Cartesian coordinates

r = magnitude of ray, A = angle (in degrees) } polar coordinates

Examples:

Find Cartesian coordinates of the point (2,30.5°) given in polar coordinates.

If a point is at (7,18) in the Cartesian system, what are its coordinates in the polar system?

A point is located at (0,-46.8). What is its location in polar coordinates?

```
:RUN
COORDINATE CONVERSION

                ( 1=CARTESIAN TO POLAR)
                (-1=POLAR TO CARTESIAN)
                ( 0=END PROGRAM)
WHICH DIRECTION? -1
R,A? 2,30.5
X = 1.72  ,  Y = 1.02
WHICH DIRECTION? 1
X,Y? 7,18
R = 19.31  ,  A = 68.75
WHICH DIRECTION? 1
X,Y? 0,-46.8
R = 46.8  ,  A = 270
WHICH DIRECTION? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "COORDINATE CONVERSION"
 20  PRINT
 30  PRINT "                ( 1=CARTESIAN TO POLAR)"
 40  PRINT "                (-1=POLAR TO CARTESIAN)"
 50  PRINT "                ( 0=END PROGRAM)"
 60  PRINT "WHICH DIRECTION";
 70  INPUT D
 79  REM - END PROGRAM?
 80  IF D=0  THEN  380
 89  REM - DIRECT PROGRAM TO PERFORM PROPER CONVERSION
 90  IF D=-1  THEN  320
 98  REM - CONVERT FROM CARTESIAN COORDINATES TO POLAR COORDINATES
 99  REM - ENTER CARTESIAN COORDINATES (ABSCISSA, ORDINATE)
100  PRINT "X,Y";
110  INPUT X,Y
119  REM - POINT ON Y-AXIS?
120  IF X=0  THEN  170
129  REM - POINT ON X-AXIS?
130  IF Y=0  THEN  260
139  REM - COMPUTE POLAR COORDINATES, ROUND OFF, PRINT
140  PRINT "R =";INT(SGN(X)*SQR(X↑2+Y↑2)*100+.5)/100;",";
150  PRINT "  A =";INT(ATN(Y/X)*180/3.1415927*100+.5)/100
160  GOTO   60
169  REM - POINT IS ON Y-AXIS; AT ORIGIN?
170  IF Y=0  THEN  240
180  PRINT "R =";ABS(Y);",";
189  REM - IS POINT ABOVE OR BELOW ORIGIN?
190  IF Y<0  THEN  220
200  PRINT "  A = 90"
210  GOTO   60
220  PRINT "  A = 270"
230  GOTO   60
239  REM - POINT IS AT ORIGIN
240  PRINT "R = 0,  A = 0"
250  GOTO   60
259  REM - POINT IS ON X-AXIS
260  PRINT "R =";ABS(X);",";
269  REM - IS POINT TO LEFT OR RIGHT OF ORIGIN?
270  IF X<0  THEN  300
280  PRINT "  A = 0"
290  GOTO   60
300  PRINT "  A = 180"
310  GOTO   60
318  REM - CONVERT FROM POLAR COORDIATES TO CARTESIAN COORDINATES
319  REM - ENTER POLAR COORDINATES (MAGNITUDE OF RAY, ANGLE)
320  PRINT "R,A";
330  INPUT R,A
339  REM - CONVERT FROM DEGREES TO RADIANS
340  M=(A-INT(A/360)*360)*3.1415927/180
349  REM - CALCULATE CARTESIAN COORDINATES, ROUND OFF, PRINT
350  PRINT "X =";INT(R*COS(M)*100+.5)/100;",";
360  PRINT "  Y =";INT(R*SIN(M)*100+.5)/100
370  GOTO   60
380  END
```

Coordinate Plot

This program plots points on a set of coordinate axes. You must provide the *x* - and *y* -coordinates of all points to be plotted, the endpoints of the *x* - and *y* -axes, and the increment between points on each axis.

The graph is unconventional in that its *x* -axis runs vertically while its *y* -axis runs horizontally. In addition, the axes do not necessarily intersect at zero. A reminder as to where the axes intersect is printed at the top of each graph.

The limit on the number of points plotted may be increased or decreased by altering statement 30 in the following manner:

```
30 DIM X(N+1),Y(N+1)
```

where *N* = the maximum number of points you wish to plot.

The length of the *y* -axis is limited by the width of your output device. This program tests for a length not to exceed 70 spaces. The test at statement 90 should be altered to accommodate your particular output device. For an output device with a line width of 112 characters you might enter:

```
90 IF B2<=108 THEN 120
```

Example:

The heights of twelve men and their sons are recorded in the table below. Plot the data points.

father	65	63	67	64	68	62	70	66	68	67	69	71
son	68	66	68	65	69	66	68	65	71	67	68	70

height in inches

```
:30 DIM X(13),Y(13)
:RUN
COORDINATE PLOT

X-AXIS: LEFT ENDPOINT, RIGHT ENDPOINT, INCREMENT? 62,73,.5
Y-AXIS: LOWER ENDPOINT, UPPER ENDPOINT, INCREMENT? 62,73,.25
NUMBER OF POINTS TO BE PLOTTED? 12
COORDINATES OF POINT  1 ? 65,68
                POINT  2 ? 63,66
                POINT  3 ? 67,68
                POINT  4 ? 64,65
                POINT  5 ? 68,69
                POINT  6 ? 62,66
                POINT  7 ? 70,68
                POINT  8 ? 66,65
                POINT  9 ? 68,71
                POINT  10 ? 67,67
                POINT  11 ? 69,68
                POINT  12 ? 71,70
```

```
INTERSECTION OF AXES AT ( 62 , 62 )

*****************+*********************************Y
*
*                 +
*
*            +
*
*                        +
*
*            +
*
*                    +   +
*
*                            +       +
*
*                        +
*
*                        +
*
*                                +
*
*
*
*
X
```

```
END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT  "COORDINATE PLOT"
 20  PRINT
 28  REM - DIMENSION OF X() AND Y() SHOULD BE LIMITED TO (N+1);
 29  REM - WHERE N=THE NUMBER OF POINTS BEING PLOTTED, MAX. LIMIT  99
 30  DIM X(100),Y(100)
 39  REM - INPUT INFORMATION TO SET UP AXES
 40  PRINT  "X-AXIS: LEFT ENDPOINT, RIGHT ENDPOINT, INCREMENT";
 50  INPUT A1, A2, A3
 60  PRINT  "Y-AXIS: LOWER ENDPOINT, UPPER ENDPOINT, INCREMENT";
 70  INPUT B1, B2, B3
 80  B2=(B2-B1)/B3
 88  REM - Y-AXIS TOO LONG FOR OUTPUT DEVICE? IF YES, CHANGE ENDPOINTS
 89  REM - OR INCREASE INCREMENT
 90  IF B2<=70  THEN  120
100  PRINT "Y-RANGE TOO LARGE"
110  GOTO   60
120  PRINT "NUMBER OF POINTS TO BE PLOTTED";
130  INPUT N
139  REM - NO POINTS TO PLOT? END PROGRAM
140  IF N=0  THEN 1070
149  REM - TOO MANY POINTS? IF YES, REENTER NUMBER OF POINTS
150  IF N<=99  THEN  180
160  PRINT "TOO MANY POINTS!"
170  GOTO  120
```

```
179  REM - LOOP TO INPUT X,Y COORDINATES FOR EACH POINT
180  FOR I=1  TO N
190  IF I>1  THEN  220
200  PRINT "COORDINATES OF POINT ";I;
210  GOTO  230
220  PRINT "                    POINT ";I;
230  INPUT X(I)  Y(I)
239  REM - ROUND OFF EACH X,Y TO NEAREST INCREMENT ON AXIS
240  X(I)=INT((X(I)-A1)/A3+.5)
250  Y(I)=INT((Y(I)-B1)/B3+.5)
260  NEXT I
269  REM - CALCULATE ADDITIONAL X AND Y COORDINATE
270  Y(N+1)=INT(B2+.5)+1
280  X(N+1)=INT((A2-A1)/A3+.5)+1
290  PRINT
299  REM - NOTE WHERE AXES CROSS
300  PRINT "INTERSECTION OF AXES AT (";A1;",";B1;")"
310  PRINT
319  REM - SORT COORDINATES; REORDER X(1) TO X(N) SMALLEST TO LARGEST
320  FOR J=1  TO N
330  FOR I=1  TO N-J
340  A=X(I)
350  B=Y(I)
360  C=X(I+1)
370  D=Y(I+1)
380  IF A<C  THEN  430
390  X(I)=C
400  Y(I)=D
410  X(I+1)=A
420  Y(I+1)=B
430  NEXT I
440  NEXT J
449  REM - NEXT POINT TO BE PLOTTED STORED IN T
450  T=1
459  REM - SKIP POINTS OUT OF X-POSITIVE RANGE
460  FOR P=0  TO N-1
470  IF X(P+1)>=0  THEN  490
480  NEXT P
489  REM - LOOP TO CALL UP EACH X-INCREMENT FOR LINES OF PRINT
490  FOR I=0  TO INT((A2-A1)/A3+.5)
500  T=T+P
509  REM - COUNT NUMBER OF POINTS TO BE PLOTTED ON EACH LINE IN P
510  P=0
519  REM - ALL POINTS PLOTTED?
520  IF T>N  THEN  540
529  REM - X-VALUE ON X-LINE? IF YES, TEST FOR Y
530  IF X(T)=I  THEN  590
539  REM - FIRST LINE? IF YES, Y-AXIS MUST BE PLOTTED
540  IF I=0  THEN  570
549  REM - PLOT X-AXIS
550  PRINT "*";
560  GOTO 1040
570  S=N+1
580  GOTO  920
590  FOR L=T  TO N
599  REM - NEXT POINT PLOTTED ON SAME LINE?
600  IF X(L)>X(T)  THEN  630
```

```
609  REM - COUNT POINTS TO BE PLOTTED ON EACH LINE
610  P=P+1
620  NEXT L
629  REM - PLOT ONE POINT
630  IF P=1  THEN  730
638  REM - LOOP TO SORT Y-COORDINATES WITH EQUAL X-COORDINATES;
639  REM - REORDER SMALLEST TO LARGEST
640  FOR J=1  TO P
650  FOR L=1  TO P-J
660  D=Y(T+L-1)
670  B=Y(T+L)
680  IF D<=B  THEN  710
690  Y(T+L-1)=B
700  Y(T+L)=D
710  NEXT L
720  NEXT J
730  FOR L=0  TO P-1
740  Z=Y(T+L)
749  REM - TEST FOR OUT-OF-RANGE Y-COORDINATE
750  IF Z>=0  THEN  770
760  NEXT L
769  REM - POINT TO BE PLOTTED ON X-AXIS?
770  IF I=0  THEN  910
779  REM - POINT TO BE PLOTTED ON Y-AXIS?
780  IF Z=0  THEN  800
789  REM - PLOT X-AXIS
790  PRINT "*";
800  IF L=P-1  THEN  870
810  FOR J=L  TO P-1
819  REM - TEST FOR OUT-OF-RANGE Y-COORDINATE
820  IF Z>B2  THEN 1040
829  REM - BYPASS DUPLICATE COORDINATES
830  IF Y(T+J)=Z  THEN  860
839  REM - PLOT POINT
840  PRINT TAB(Z);"+";
850  Z=Y(T+J)
860  NEXT J
869  REM - TEST FOR OUT-OF-RANGE Y-COORDINATE
870  IF Z<0  THEN 1040
880  IF Z>B2  THEN 1040
889  REM - PLOT POINT
890  PRINT TAB(Z);"+";
900  GOTO 1040
910  S=T+L
919  REM - LOOP TO ESTABLISH PRINT FOR FIRST LINE
920  FOR J=0  TO B2
929  REM - POINT TO BE PLOTTED?
930  IF Y(S)<>J  THEN 1010
939  REM - PLOT POINT
940  PRINT "+";
949  REM - BYPASS DUPLICATE COORDINATES
950  FOR K=S  TO T+P-1
960  IF Y(K)=Y(S)  THEN  990
970  S=K
980  GOTO 1020
990  NEXT K
1000 GOTO 1020
```

```
1009  REM - PLOT Y-AXIS
1010  PRINT "*";
1020  NEXT J
1029  REM - LABEL Y-AXIS
1030  PRINT "Y";
1039  REM - ADVANCE OUTPUT DEVICE TO NEXT LINE
1040  PRINT
1050  NEXT I
1059  REM - LABEL X-AXIS
1060  PRINT "X"
1070  END
```

Plot of Polar Equation

This program plots a given function in polar coordinates. There are up to 90 points plotted, one every four degrees. (Some points may overlap.)

The graph is conventional in that the x -axis runs horizontally, the y -axis runs vertically, and they intersect at zero. You need only specify the absolute value of the endpoints.

The increment between each point on the x - and y -axes is adjusted so that a value of one on either axis is equidistant from zero. This allows the function to be plotted with minimal distortion. An adjustment of each increment is necessary because of different spacing horizontally and vertically on an output device. (This program assumes ten spaces per inch horizontally and six spaces per inch vertically. If your output device differs, the graph may be distorted.)

It is necessary for you to enter the function to be plotted before you run the program. The function must be entered as a function of d. $f(d)$ will be entered and set equal to F at line 130. For example, the function $f(d) = 2*(1-\cos(d))$ will be entered as follows:

```
130 F=2*(1-COS(D))
```

(Continued on next page)

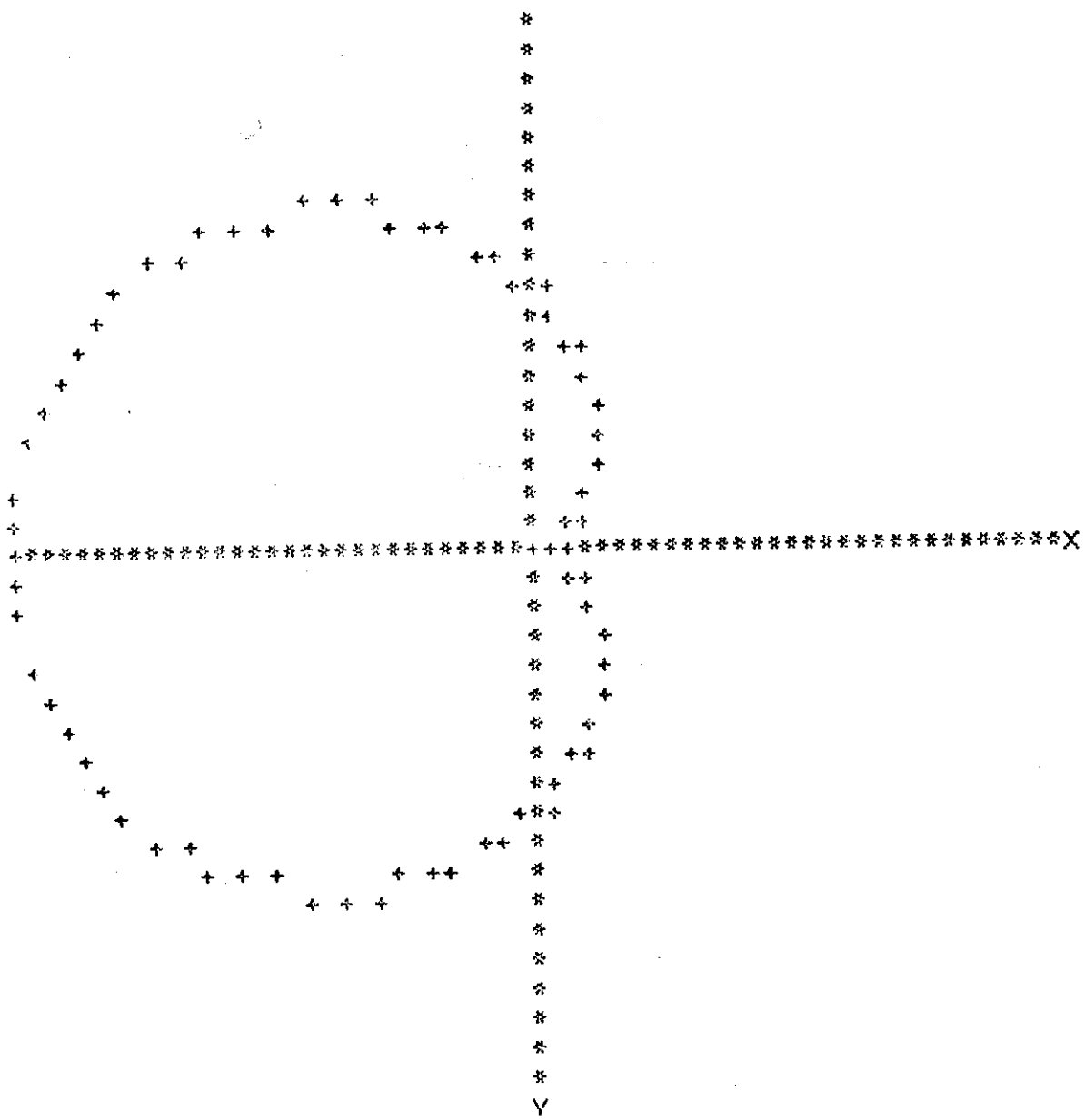
Example:

Plot the equation $f(d) = 2 \cdot (1-\cos(d))$.

```
:130 F=2*(1-COS(D))
:RUN
PLOT OF POLAR EQUATION

ABSOLUTE VALUE OF ENDPOINTS? 4

INCREMENT OF X-AXIS = .1333333333333
INCREMENT OF Y-AXIS = .2222222222222
```




```
END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "PLOT OF POLAR EQUATION"
 20  PRINT
 28  REM - COORDINATE ARRAYS SET FOR 90 POINTS;
 29  REM - ONE EXTRA X-COORDINATE IS CALCULATED IN PROGRAM
 30  DIM X(91),Y(90)
 39  REM - NUMBER OF POINTS TO BE CALCULATED
 40  N=90
 49  REM - ABSOLUTE VALUE OF ALL ENDPOINTS ARE EQUAL
 50  PRINT "ABSOLUTE VALUE OF ENDPOINTS";
 60  INPUT Z
 70  PRINT
 79  REM - CALCULATE INCREMENTS OF AXES ACCORDING TO CHARACTERS PER AX
   IS
 80  PRINT "INCREMENT OF X-AXIS =";Z/30
 90  PRINT "INCREMENT OF Y-AXIS =";Z/18
100  PRINT
110  FOR I=1  TO N
119  REM - CONVERT DEGREES TO RADIANS
120  D=.06981317*I
130  REM - ENTER FUNCTION HERE (F="FUNCTION")
139  REM - CALCULATE EACH CARTESIAN COORDINATE, ROUND OFF TO NEAREST I
   NCREMENT ON AXIS
140  X(I)=INT(((F*COS(D)/Z+1)*30)+.5)
150  Y(I)=INT(((-F*SIN(D)/Z+1)*18)+.5)
160  NEXT I
169  REM - SORT COORDINATES; REORDER Y(1) TO Y(N) SMALLEST TO LARGEST
170  FOR J=1  TO N
180  FOR I=1  TO N-J
190  A=X(I)
200  B=Y(I)
210  IF B<=Y(I+1)  THEN  260
220  X(I)=X(I+1)
230  Y(I)=Y(I+1)
240  X(I+1)=A
250  Y(I+1)=B
260  NEXT I
270  NEXT J
279  REM - NEXT POINT TO BE PLOTTED STORED IN T
280  T=1
289  REM - SKIP POINTS OUT OF Y-POSITIVE RANGE
290  FOR P=0  TO N-1
300  IF Y(P+1)>=0  THEN  320
310  NEXT P
319  REM - LOOP TO CALL UP EACH Y-INCREMENT FOR LINES OF PRINT
320  FOR I=0  TO 36
330  T=T+P
339  REM - NUMBER OF POINTS TO BE PLOTTED ON EACH LINE STORED IN P
340  P=0
349  REM - ALL POINTS PLOTTED?
350  IF T>N  THEN  370
359  REM - Y-VALUE ON Y-LINE?
360  IF Y(T)=I  THEN  420
369  REM - PRINT X-AXIS?
370  IF I=18  THEN  400
379  REM - PRINT Y-AXIS
```

```
380  PRINT TAB(30);"*";
390  GOTO  860
400  S=N+1
410  GOTO  740
420  FOR L=T  TO N
429  REM - NEXT POINT TO BE PLOTTED ON SAME LINE?
430  IF Y(L)>Y(T)  THEN  450
440  P=P+1
450  NEXT L
460  IF P=1  THEN  560
468  REM - LOOP TO SORT X-COORDINATES WITH EQUAL Y-COORDINATES;
469  REM - REORDER SMALLEST TO LARGEST
470  FOR J=1  TO P
480  FOR L=1  TO P-J
490  C=X(T+L-1)
500  A=X(T+L)
510  IF C<=A  THEN  540
520  X(T+L-1)=A
530  X(T+L)=C
540  NEXT L
550  NEXT J
559  REM - PRINT X-AXIS?
560  IF I=18  THEN  730
570  L=-1
580  S=0
590  FOR K=0  TO P-1
599  REM - MORE THAN ONE POINT TO BE PLOTTED AT SAME POINT ON GRAPH?
600  IF X(T+K)=L  THEN  690
610  L=X(T+K)
619  REM - PLOT POINT ON Y-AXIS?
620  IF L=30  THEN  660
629  REM - PLOT POINT TO THE LEFT OF Y-AXIS?
630  IF L<30  THEN  670
640  IF S=1  THEN  670
649  REM - PRINT Y-AXIS
650  PRINT TAB(30);"*";
660  S=1
669  REM - POINT OUTSIDE OF POSITIVE X-RANGE?
670  IF L>60  THEN  860
679  REM - PLOT POINT
680  PRINT TAB(L);"+";
690  NEXT K
700  IF S=1  THEN  860
709  REM - PRINT Y-AXIS
710  PRINT TAB(30);"*";
720  GOTO  860
730  S=T
739  REM - LOOP TO PRINT LINE OF X-AXIS
740  FOR J=0  TO 60
750  IF X(S)<>J  THEN  830
759  REM - PLOT POINT ON X-AXIS
760  PRINT "+";
770  FOR K=S  TO T+P-1
780  IF X(K)=X(S)  THEN  810
790  S=K
800  GOTO  840
810  NEXT K
```

```
820  GOTO  840
829  REM - PRINT X-AXIS
830  PRINT "*";
840  NEXT J
849  REM - LABEL X-AXIS
850  PRINT "X";
860  PRINT
870  NEXT I
879  REM - LABEL Y-AXIS
880  PRINT TAB(30);"Y"
890  END
```

Plot of Functions

This program calculates and plots up to nine functions. All functions must be functions of x, and all will be plotted on the same set of axes.

To set up the axes you must input the endpoints of the x - and y -axes. You must also state the increment by which the points on each axis are to be increased.

The graph is unconventional in that its x -axis runs vertically while its y -axis runs horizontally. To read the graph you must either turn your output 90° counterclockwise or mentally adjust to the change in convention.

The graph is also unconventional in that its axes do not necessarily cross at zero. A reminder as to where the axes cross is printed at the top of each graph.

You must enter the functions to be plotted as program statements prior to running the program. Statement numbers 221 to 229 are reserved for this purpose. Functions must be entered in the number sequence Y(1), Y(2),. . . Y(9). For example, if you wish to plot the functions $f(x) = 2x + 1$ and $f(x) = \sqrt{x}$, you must type:

```
221 Y(1)=2*X+1
222 Y(2)=SQR(X)
```

The length of the y -axis is limited by the width of your output device. This program tests for a length not to exceed 70 spaces. The test at statement 140 should be altered to accommodate your particular output device. For example, an output device with a line width of 64 characters would accommodate a graph 62 spaces wide. You would change statement 140 to:

```
140 IF Y2<=62 THEN 170
```

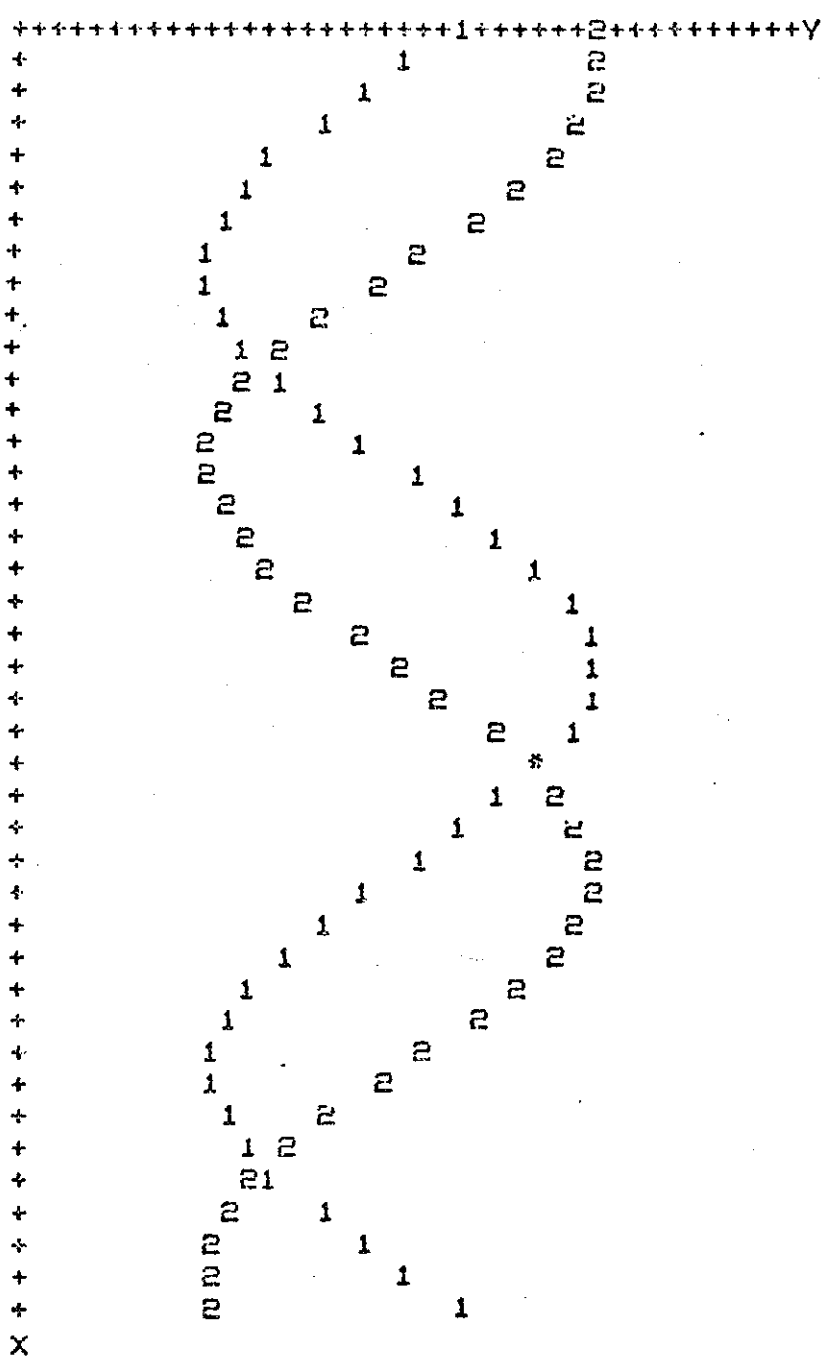
Example:

Plot the equations $f(x) = \cos(x)$ and $f(x) = \sin(x)$.

```
:221 Y(1)=COS(X)
:222 Y(2)=SIN(X)
:RUN
PLOT OF FUNCTIONS

NUMBER OF FUNCTIONS TO BE PLOTTED? 2
X-AXIS:LEFT ENDPOINT,RIGHT ENDPOINT,INCREMENT? -5,5,.25
Y-AXIS:LOWER ENDPOINT,UPPER ENDPOINT,INCREMENT? -2,2,.1
```

```
X-AXIS CROSSES Y-AXIS AT Y=-2
Y-AXIS CROSSES X-AXIS AT X=-5
```




```
END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "PLOT OF FUNCTIONS"
 20  PRINT
 29  REM - NUMBER OF FUNCTIONS WHICH CAN BE PLOTTED IS LIMITED TO 9
 30  DIM Y(9),A$(11)
 40  FOR I=1  TO 11
 49  REM - GET VALUES FOR A$-ARRAY FROM DATA TABLE AT STATEMENT 470
 50  READ A$(I)
 60  NEXT I
 69  REM - STATEMENTS 70 TO 120 REQUEST USER INPUT
 70  PRINT "NUMBER OF FUNCTIONS TO BE PLOTTED";
 80  INPUT N
 90  PRINT "X-AXIS:LEFT ENDPOINT,RIGHT ENDPOINT,INCREMENT";
100  INPUT X1,X2,X3
110  PRINT "Y-AXIS:LOWER ENDPOINT,UPPER ENDPOINT,INCREMENT";
120  INPUT Y1,Y2,Y3
129  REM - CALCULATE NUMBER OF SPACES ON Y-AXIS
130  Y2=(Y2-Y1)/Y3
138  REM - TEST FOR A Y-AXIS TOO LONG FOR OUTPUT DEVICE.  IF YES, THEN
139  REM - LESSEN RANGE OR INCREASE INCREMENT
140  IF Y2<=70  THEN  170
150  PRINT "Y-RANGE TOO LARGE"
160  GOTO  110
170  PRINT
180  PRINT
189  REM - MAKE NOTE OF WHERE AXES CROSS
190  PRINT "X-AXIS CROSSES Y-AXIS AT Y=";Y1
200  PRINT "Y-AXIS CROSSES X-AXIS AT X=";X1
210  PRINT
219  REM - SET UP LOOP TO READ VALUE AT EACH X-INCREMENT
220  FOR X=X1  TO X2  STEP X3
221  REM - FUNCTIONS Y(1) TO Y(9) SHOULD BE ENTERED AT LINES 221 TO 229
230  FOR I=1  TO N
239  REM - ESTABLISH THE ROUNDED VALUE OF Y FOR EACH X-INCREMENT VALUE
240  Y(I)=INT((Y(I)-Y1)/Y3+.5)
250  NEXT I
259  REM - LOOP TO READ VALUE OF EACH Y-INCREMENT
260  FOR I=0  TO Y2
269  REM - S COUNTS THE NUMBER OF VALUES AT EACH Y-INCREMENT FOR EACH X
270  S=0
280  FOR J=1  TO N
289  REM - PLOT A POINT ON THIS SPOT? IF YES, STORE FUNCTION NUMBER IN T
290  IF Y(J)<>I  THEN  320
300  S=S+1
310  T=J
320  NEXT J
327  REM - TEST FOR NUMBER OF POINTS TO PLOT ON EACH SPOT;
329  REM - IF 0 PRINT "+" (FIRST LINE ONLY), IF 1 PRINT FUNCTION NUMBER,
   IF 2 OR MORE PRINT "*"
330  IF S>0  THEN  360
340  PRINT A$(SGN(I)+10);
350  GOTO  400
360  IF S>1  THEN  390
370  PRINT A$ (T);
380  GOTO  400
390  PRINT "*";
```

```
400  NEXT I
409  REM - LABEL AXES AT THE LAST SPACE ON EACH AXIS
410  IF X>X1  THEN  430
420  PRINT "Y";
429  REM - ADVANCE PRINTER TO NEXT LINE
430  PRINT
439  REM - PRINT SPACE INSTEAD OF "+" AFTER FIRST LINE OF PRINT (Y-AX
440  A$(11)=" "
450  NEXT X
460  PRINT "X"
470  DATA "1","2","3","4","5","6","7","8","9","+","+"
480  END
```

Linear Interpolation

This program calculates the y-coordinates of points on a line given their x-coordinates. It is necessary to know coordinates of two points on the same line.

The point is interpolated using the following formula:

$$y = y_1 + \frac{(y_2 - y_1) \cdot (x - x_1)}{(x_2 - x_1)}$$

where: x_1, y_1 = coordinates of first point on the line
x_2, y_2 = coordinates of second point on the line
x = abscissa of point to be interpolated
y = ordinate of the point on the line with x

Examples:

A conversion table lists 60°F as 15.56°C and 90°F as 32.22°C. Calculate degrees Celsius of 73°F and 85.6°F.

A new sales tax of 17.5% has been imposed on us. What will be the tax on a sofa which sells for $455.68?

```
:RUN
LINEAR INTERPOLATION

X,Y OF FIRST POINT? 60,15.56
X,Y OF SECOND POINT? 90,32.22
INTERPOLATE: X =? 73
             Y =  22.779

MORE POINTS ON THIS LINE (1=YES, 0=NO)? 1

INTERPOLATE: X =? 85.6
             Y =  29.777

MORE POINTS ON THIS LINE (1=YES, 0=NO)? 0

NEW LINE (1=YES, 0=NO)? 1

X,Y OF FIRST POINT? 0,0
X,Y OF SECOND POINT? 100,17.5
INTERPOLATE: X =? 455.68
             Y =  79.744

MORE POINTS ON THIS LINE (1=YES, 0=NO)? 0

NEW LINE (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "LINEAR INTERPOLATION"
 20  PRINT
 29  REM - ENTER X- AND Y-COORDINATES OF TWO POINTS ON THE LINE
 30  PRINT "X,Y OF FIRST POINT";
 40  INPUT X1,Y1
 50  PRINT "X,Y OF SECOND POINT";
 60  INPUT X2,Y2
 69  REM - ENTER X-COORDINATE OF POINT TO BE INTERPOLATED
 70  PRINT "INTERPOLATE: X =";
 80  INPUT X
 89  REM - COMPUTE CORRESPONDING Y-COORDINATE
 90  Y=Y1+(Y2-Y1)/(X2-X1)*(X-X1)
 99  REM - ROUND OFF, PRINT
100  PRINT "              Y = ";INT(Y*1000+.5)/1000
110  PRINT
120  PRINT "MORE POINTS ON THIS LINE (1=YES, 0=NO)";
130  INPUT Z
140  PRINT
150  IF Z=1  THEN   70
159  REM - INTERPOLATE ON ANOTHER LINE?
160  PRINT "NEW LINE (1=YES, 0=NO)";
170  INPUT Z
180  IF Z=1  THEN   20
190  END
```

Curvilinear Interpolation

This program computes y-coordinates of points on a curve given their x-coordinates. You must input ...inates of known points on the curve, no two having the same abscissa.

The computations are performed using the Lagrange method of interpolation.

The number of known points on the curve which may be entered in the program is limited to 50. You may ...ease or decrease this limit by altering statement 30 according to the following scheme:

```
30 DIM X(P), Y(P)
```

where P = the number of known points on a curve.

...amples:

Consider the curve $y = x^3 - 3x + 3$. You know that the points (-3,-15), (-2,1), (-1,5), (0,3), (1,1), (2,5), and (...) are on the curve. What is the value of y when $x = -1.65$ and 0.2?

Given the following points from a sine curve, what is the sine of -2.47 and the sine of 1.5?

(-5,.958)	(0,0)
(-4,.757)	(1,.841)
(-3,-.141)	(2,.909)
(-2,-.909)	(3,.141)
(-1,-.841)	(4,-.757)
	(5,-.959)

```
30 DIM X(11), Y(11)
RUN
URVILINEAR INTERPOLATION

UMBER OF KNOWN POINTS? 7
,Y OF POINT 1 ? -3,-15
,Y OF POINT 2 ? -2,1
,Y OF POINT 3 ? -1,5
,Y OF POINT 4 ? 0,3
,Y OF POINT 5 ? 1,1
,Y OF POINT 6 ? 2,5
,Y OF POINT 7 ? 3,21

NTERPOLATE: X=? -1.65
            Y= 3.45787499999

ORE X ON THIS CURVE (1=YES, 0=NO)? 1

NTERPOLATE: X=? .2
            Y= 2.40800000002

ORE X ON THIS CURVE (1=YES, 0=NO)? 0
ORE X ON ANOTHER CURVE (1=YES, 0=NO)? 1

UMBER OF KNOWN POINTS? 11
,Y OF POINT 1 ? -5,.958
,Y OF POINT 2 ? -4,.757
,Y OF POINT 3 ? -3,-.141
```

```
X,Y OF POINT 4 ? -2,-.909
X,Y OF POINT 5 ? -1,-.841
X,Y OF POINT 6 ? 0,0
X,Y OF POINT 7 ? 1,.841
X,Y OF POINT 8 ? 2,.909
X,Y OF POINT 9 ? 3,.141
X,Y OF POINT 10 ? 4,-.757
X,Y OF POINT 11 ? 5,-.959

INTERPOLATE: X=? -2.47
             Y=-.6218395970637

MORE X ON THIS CURVE (1=YES, 0=NO)? 1

INTERPOLATE: X=? 1.5
             Y= .9971637992869

MORE X ON THIS CURVE (1=YES, 0=NO)? 0
MORE X ON ANOTHER CURVE (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "CURVILINEAR INTERPOLATION"
 20  PRINT
 28  REM - LIMIT X() AND Y() TO MAXIMUM NUMBER OF POINTS KNOWN ON ANY
 25  REM - CURVE TO BE ENTERED
 30  DIM X(50), Y(50)
 40  PRINT "NUMBER OF KNOWN POINTS";
 50  INPUT P
 60  FOR I=1  TO P
 69  REM - ENTER COORDINATES OF KNOWN POINTS ON CURVE
 70  PRINT "X,Y OF POINT";I;
 80  INPUT X(I),Y(I)
 90  NEXT I
100  PRINT
109  REM - ENTER X-COORDINATE OF POINT TO BE INTERPOLATED
110  PRINT "INTERPOLATE: X=";
120  INPUT A
130  B=0
138  REM - COMPUTE CORRESPONDING Y-COORDINATES BY LAGRANGE METHOD OF
139  REM - INTERPOLATION
140  FOR J=1  TO P
150  T=1
160  FOR I=1  TO P
170  IF I=J  THEN  190
180  T=T*(A-X(I))/(X(J)-X(I))
190  NEXT I
200  B=B+T*Y(J)
210  NEXT J
219  REM - PRINT RESULTS
220  PRINT "                Y=";B
230  PRINT
239  REM - INTERPOLATE  MORE POINTS ON SAME CURVE?
240  PRINT "MORE X ON THIS CURVE (1=YES, 0=NO)";
```

```
250   INPUT C
260   IF C=1   THEN   100
269   REM - RESTART OR END PROGRAM?
270   PRINT "MORE X ON ANOTHER CURVE (1=YES, 0=NO)";
280   INPUT C
290   IF C=1   THEN    20
300   END
```

Integration: Simpson's Rule

This program approximates the definite integral of a function. The integral is computed using Simpson's rule.

The method the program takes is optional: you must supply either the function of the curve or values of the function at specified intervals. For both methods you must enter the limits of integration and the increment between points within the limits.

If the function to be integrated is known, it must be entered before running the program. The function will be defined at line 50. For example, the function $f(x) = x^3$ will be entered as follows:

```
50 DEFFNC(X)=X↑3
```

Examples:

Find the definite integral of the function $f(x) = x^3$ between 0 and 2 with increments of .2 and .1.

What is the integral of a curve between -1 and 1 if the points known are as follows:

(-1,.54)	(.25,.969)
(-.75,.73)	(.5,.878)
(-.5,.878)	(.75,.73)
(-.25,.969)	(1,.54)
(0,1)	

```
:50 DEFFNC(X)=X↑3
:RUN
INTEGRATION: SIMPSON'S RULE

SELECTION: 1=KNOWN FORMULA, 0=UNKNOWN FORMULA? 1
LOWER, UPPER LIMIT OF INTEGRATION? 0,2
INCREMENT OF X? .2
INTEGRAL IS 4

END PROGRAM

:RUN
INTEGRATION: SIMPSON'S RULE

SELECTION: 1=KNOWN FORMULA, 0=UNKNOWN FORMULA? 1
LOWER, UPPER LIMIT OF INTEGRATION? 0,2
INCREMENT OF X? .1
INTEGRAL IS 4

END PROGRAM

:RUN
INTEGRATION: SIMPSON'S RULE

SELECTION: 1=KNOWN FORMULA, 0=UNKNOWN FORMULA? 0
LOWER, UPPER LIMIT OF INTEGRATION? -1,1
INCREMENT OF X? .25
FIRST, LAST VALUE OF F(X)? .54,.54
```

```
VALUE OF F(X) AT INTERVAL 1 (X=-.75 )? .73
VALUE OF F(X) AT INTERVAL 2 (X=-.5 )? .878
VALUE OF F(X) AT INTERVAL 3 (X=-.25 )? .969
VALUE OF F(X) AT INTERVAL 4 (X= 0 )? 1
VALUE OF F(X) AT INTERVAL 5 (X= .25 )? .969
VALUE OF F(X) AT INTERVAL 6 (X= .5 )? .878
VALUE OF F(X) AT INTERVAL 7 (X= .75 )? .73
INTEGRAL IS 1.682

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "INTEGRATION: SIMPSON'S RULE"
 20  PRINT
 30  PRINT "SELECTION: 1=KNOWN FORMULA, 0=UNKNOWN FORMULA";
 40  INPUT S
 49  REM - IF FUNCTION IS KNOWN ENTER AT LINE 50 (DEFFNC(X)="FUNCTION")
 50  DEFFNC(X)=X
 60  PRINT "LOWER, UPPER LIMIT OF INTEGRATION";
 70  INPUT A,B
 80  PRINT "INCREMENT OF X";
 90  INPUT X1
 98  REM - INCREMENT MUST DIVIDE INTERVAL INTO EQUAL SUBINTERVALS;
 99  REM - IF NOT, CHANGE INCREMENT
100  IF (B-A)/X1<>INT((B-A)/X1)  THEN  80
110  IF S=1  THEN  150
119  REM - FORMULA NOT KNOWN; ENTER FUNCTION VALUE AT INTEGRATION LIMITS
120  PRINT "FIRST, LAST VALUE OF F(X)";
130  INPUT Y1,Y2
140  GOTO  170
149  REM - FORMULA KNOWN; CALCULATE F(X) AT INTEGRATION LIMITS
150  Y1=FNC(A)
160  Y2=FNC(B)
170  C=0
180  D=0
189  REM - LOOP FOR EACH SUBINTERVAL
190  FOR I=1  TO (B-A)/X1-.5
200  IF S=1  THEN  240
209  REM - ENTER KNOWN FUNCTION VALUE AT EACH INTERVAL
210  PRINT "VALUE OF F(X) AT INTERVAL";I;"(X=";A+I*X1;")";
220  INPUT Y
230  GOTO  250
239  REM - CALCULATE F(X) AT EACH SUBINTERVAL
240  Y=FNC(A+I*X1)
249  REM - INTERVAL EVEN OR ODD?
250  IF I/2=INT(I/2)  THEN  280
259  REM - SUM ALL ODD-INTERVAL FUNCTION VALUES
260  C=C+Y
270  GOTO  290
279  REM - SUM ALL EVEN-INTERVAL FUNCTION VALUES
280  D=D+Y
290  NEXT I
299  REM - COMPUTE INTEGRAL, PRINT
300  PRINT "INTEGRAL IS";X1/3*(Y1+4*C+2*D+Y2)
310  END
```

Integration: Trapezoidal Rule

This program approximates the definite integral of a function. The integral is computed using the trapezoidal rule. You must provide the limits of integration and the number of intervals within the limits.

The function to be integrated must be entered before running the program. The function of x will be defined at line 30. For example, the function $f(x) = x^3$ will be entered as follows:

```
30 DEFFNC(X)=X↑3
```

Examples:

Find the definite integral of the function $f(x) = x^3$ between 0 and 2 with 10 and 20 intervals.

Find the definite integral of the function $f(x) = x^{-2}$ between 1 and 2 and 2 and 3 using 10 subintervals.

```
:30 DEFFNC(X)=X↑3
:RUN
INTEGRATION: TRAPEZOIDAL RULE

(ENTER 0,0 TO END PROGRAM)
INTEGRATION LIMITS (LOWER, UPPER)? 0,2
NUMBER OF INTERVALS? 10
INTEGRAL = 4.04

INTEGRATION LIMITS (LOWER, UPPER)? 0,2
NUMBER OF INTERVALS? 20
INTEGRAL = 4.01

INTEGRATION LIMITS (LOWER, UPPER)? 0,0

END PROGRAM


:30 DEFFNC(X)=1/X↑2
:RUN
INTEGRATION: TRAPEZOIDAL RULE

(ENTER 0,0 TO END PROGRAM)
INTEGRATION LIMITS (LOWER, UPPER)? 1,2
NUMBER OF INTERVALS? 10
INTEGRAL = .5014551274644

INTEGRATION LIMITS (LOWER, UPPER)? 2,3
NUMBER OF INTERVALS? 10
INTEGRAL = .16681318133

INTEGRATION LIMITS (LOWER, UPPER)? 0,0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "INTEGRATION: TRAPEZOIDAL RULE"
 20  PRINT
 30  REM - ENTER FUNCTION HERE (DEFFNC(X)="FUNCTION")
 40  PRINT "(ENTER 0,0 TO END PROGRAM)"
 50  PRINT "INTEGRATION LIMITS (LOWER, UPPER)";
 60  INPUT A,B
 69  REM - END PROGRAM?
 70  IF A=B  THEN  190
 80  PRINT "NUMBER OF INTERVALS";
 90  INPUT N
100  I=0
109  REM - D IS THE SIZE OF EACH INTERVAL
110  D=(B-A)/N
119  REM - ADD UP THE AREA OF EACH TRAPEZOID
120  FOR J=A  TO B  STEP D
130  I=I+FNC(J)
140  NEXT J
149  REM - COMPUTE INTEGRAL, PRINT
150  I=(I-(FNC(A)+FNC(B))/2)*D
160  PRINT "INTEGRAL =";I
170  PRINT
180  GOTO   50
190  END
```

Integration: Gaussian Quadrature

This program approximates the definite integral of a function. You must provide the limits of integration and the number of intervals within the limits.

The interval of integration is divided into equal subintervals. The definite integral is computed over each subinterval using Gauss' formula. The integrals of the subintervals are summed to give the definite integral of the full interval.

You must enter the function to be integrated before running the program. The function of x will be defined at line 30. For example, the function $f(x) = x^3$ will be entered as follows:

```
30 DEFFNC(X)=X↑3
```

Examples:

Find the definite integral of the function $f(x) = x^3$ between 0 and 2 with 10 and 20 subintervals.

Find the definite integral of the function $f(x) = x^{-2}$ between 1 and 2 and 3 using 10 subintervals.

```
:30 DEFFNC(X)=X↑3
:RUN
INTEGRATION: GAUSSIAN QUADRATURE

INTEGRATION LIMITS (LOWER,UPPER)? 0,2
NUMBER OF INTERVALS? 10
INTEGRAL = 4.000000027887

CHANGE DATA AND RECOMPUTE?
(0=NO, 1=NEW INTEGRATION LIMITS, 2=NEW NO. OF INTERVALS)? 2
NUMBER OF INTERVALS? 20
INTEGRAL = 4.000000027968

CHANGE DATA AND RECOMPUTE?
(0=NO, 1=NEW INTEGRATION LIMITS, 2=NEW NO. OF INTERVALS)? 0

END PROGRAM


:30 DEFFNC(X)=1/X↑2
:RUN
INTEGRATION: GAUSSIAN QUADRATURE

INTEGRATION LIMITS (LOWER,UPPER)? 1,2
NUMBER OF INTERVALS? 10
INTEGRAL = .5000000034951

CHANGE DATA AND RECOMPUTE?
(0=NO, 1=NEW INTEGRATION LIMITS, 2=NEW NO. OF INTERVALS)? 1
INTEGRATION LIMITS (LOWER,UPPER)? 2,3
NUMBER OF INTERVALS? 10
INTEGRAL = .166666678324
```

```
NGE DATA AND RECOMPUTE?
=0, 1=NEW INTEGRATION LIMITS, 2=NEW NO. OF INTERVALS)? 0

PROGRAM


GRAM LISTING

 10  PRINT "INTEGRATION: GAUSSIAN QUADRATURE"
 20  PRINT
 30  REM - ENTER FUNCTION HERE (DEFFNC(X)="FUNCTION")
 39  REM - ABSCISSAS AND WEIGHT FACTORS FOR 20-POINT GUASSIAN INTEGRAT
   ION
 40  DATA .076526521,.15275339,.22778585,.14917299,.37370609
 50  DATA .14209611,.510867,.13168864,.63605368,.11819453
 60  DATA .74633191,.10193012,.83911697,.083276742,.91223443
 70  DATA .062672048,.96397193,.04060143,.9931286,.017614007
 80  PRINT "INTEGRATION LIMITS (LOWER,UPPER)";
 90  INPUT X,Y
100  PRINT "NUMBER OF INTERVALS";
110  INPUT N
120  S=(Y-X)/N/2
130  T=X+S
140  R=0
145  REM - COMPUTE INTEGRAL FOR EACH SUBINTERVAL
150  FOR I=1  TO N
160  P=0
169  REM - COMPUTE SUMMATION FACTOR FOR EACH SUBINTERVAL
170  FOR J=1  TO 10
180  READ A,B
190  P=P+B*(FNC(S*A+T)+FNC(T-S*A))
200  NEXT J
210  RESTORE
220  R=R+P*S
230  T=T+2*S
240  NEXT I
250  PRINT "INTEGRAL =";R
260  PRINT
270  PRINT "CHANGE DATA AND RECOMPUTE?"
280  PRINT "(0=NO, 1=NEW INTEGRATION LIMITS, 2=NEW NO. OF INTERVALS)";
290  INPUT S
300  IF S=1  THEN   80
310  IF S=2  THEN  100
320  END
```

Derivative

This program calculates the derivative of a given function at a given point.

You must enter the function being evaluated before you run the program. The function will be entered in a definition statement at line 30. For example, to evaluate the equation $f(x) = x^2 + \cos(x)$ you would enter the following:

```
30 DEFFNC(X)=X↑2+COS(X)
```

Example:

Calculate the derivative of the equation $x^2 + \cos(x) = 0$ when $x = -1$, $x = 0$, and $x = 1$.

```
:30 DEFFNC(X)=X↑2+COS(X)
:RUN
DERIVATIVE

(ENTER X=99999 TO END PROGRAM)
DERIVATIVE AT X=? -1
                    IS-1.1585287562224
DERIVATIVE AT X=? 0
                    IS 1.53600000E-09
DERIVATIVE AT X=? 1
                    IS 1.1585287976960
DERIVATIVE AT X=? 99999

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "DERIVATIVE"
 20  PRINT
 30  REM - ENTER DEFFNC(X) HERE
 40  PRINT "(ENTER X=99999 TO END PROGRAM)"
 50  PRINT "DERIVATIVE AT X=";
 60  INPUT X1
 69  REM - TEST FOR END OF PROGRAM
 70  IF X1=99999  THEN  160
 80  D=0
 89  REM - CALCULATE DIFFERENCE QUOTIENTS FOR POINTS APPROACHING X
 90  FOR N=1  TO 10
100  D1=D
110  X=X1+.5↑N
120  D=(FNC(X)-FNC(X1))/(X-X1)
130  NEXT N
139  REM - APPROXIMATE DERIVATIVE OF FUNCTION AT X, PRINT
140  PRINT "                          IS";2*D-D1
149  REM - RESTART PROGRAM
150  GOTO   50
160  END
```

Roots of Quadratic Equations

This program calculates the roots of a quadratic equation. The equation must be in the following form:

$$ax^2 + bx + c = 0$$

where a,b,c are real coefficients.

The formula used to calculate the roots is:

$$\text{root} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Example:

Compute the roots of the following equations:

$$2x^2 + x - 1 = 0$$
$$x^2 + 4x + 6 = 0$$

```
;RUN
ROOTS OF QUADRATIC EQUATIONS

COEFFICIENTS A,B,C? 2,1,-1
ROOTS (REAL):  -1  ,  .5

MORE DATA (1=YES, 0=NO)? 1

COEFFICIENTS A,B,C? 1,4,6
ROOTS (COMPLEX):  -2  + OR - 1.41421356235  I

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "ROOTS OF QUADRATIC EQUATIONS"
 20  PRINT
 29  REM - ENTER COEFFICIENTS A,B,C OF A*X↑2 + B*X + C
 30  PRINT "COEFFICIENTS A,B,C";
 40  INPUT A,B,C
 50  S=B↑2-4*A*C
 60  R=SQR(ABS(S))
 69  REM - COMPLEX ROOTS?
 70  IF S<0  THEN  100
 79  REM - CALCULATE ROOTS, LABEL, PRINT
 80  PRINT "ROOTS (REAL):  ";(-B-R)/(2*A);",  ";(-B+R)/(2*A)
 90  GOTO  110
100  PRINT "ROOTS (COMPLEX):  ";-B/(2*A);" + OR -";R/(2*A);" I"
110  PRINT
119  REM - RESTART OR END PROGRAM?
```

```
120   PRINT "MORE DATA (1=YES, 0=NO)";
130   INPUT X
140   IF X=1   THEN    20
150   END
```

Real Roots of Polynomials: Newton

This program calculates real roots of a polynomial with real coefficients. You must give an estimate of each root.

The calculations are performed using Newton's method for approximating roots of equations. The value of the error and derivative are included for each root calculated.

The equation you enter is presently limited to a degree of 10. You may enter a larger degree of equation by altering statements 30 and 40 of the program according to the following scheme:

```
30 DIM A(N+1),B(N+1)
40 FOR I=1 TO N+1
```

where N = degree of equation.

Example:

Find the roots of $4x^4 - 2.5x^2 - x + 0.5$

```
:RUN
REAL ROOTS OF POLYNOMIALS: NEWTON

DEGREE OF EQUATION? 4
COEFFICIENT A( 0 )? .5
COEFFICIENT A( 1 )? -1
COEFFICIENT A( 2 )? -2.5
COEFFICIENT A( 3 )? 0
COEFFICIENT A( 4 )? 4

GUESS? -.8

 ROOT              ERROR              DERIVATIVE
 .3035763402058 -1.40000000E-13 -2.07024700045 3

ANOTHER VALUE (1=YES, 0=NO)? 0
ANOTHER FUNCTION (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "REAL ROOTS OF POLYNOMIALS: NEWTON"
 20  PRINT
 28  REM - LIMIT A() AND B() TO N+1; WHEN THIS IS DONE, LOOP AT LINE 40
 29  REM - SHOULD BE SET TO TEST FROM 1 TO N+1
 30  DIM A(11),B(11)
 39  REM - INITIALIZE ARRAY VARIABLES
 40  FOR I=1  TO 11
 50  A(I)=0
 60  B(I)=0
 70  NEXT I
 80  PRINT "DEGREE OF EQUATION";
 90  INPUT N
100  FOR I=1  TO N+1
109  REM - ENTER COEFFICIENTS IN ORDER OF LESSER TO HIGHER DEGREE
```

```
110  PRINT "COEFFICIENT A(";I-1;")";
120  INPUT A(I)
130  NEXT I
140  FOR I=1  TO 10
149  REM - CALCULATE COEFFICIENT OF DERIVATIVE OF POLYNOMIAL
150  B(I)=A(I+1)*I
160  NEXT I
170  PRINT
179  REM - INITIALIZE GUESS
180  PRINT "GUESS";
190  INPUT X
200  Q=0
210  S=1
220  F1=0
230  F0=0
239  REM - COUNT ITERATIONS
240  Q=Q+1
250  FOR I=1  TO N+1
259  REM - CALCULATE VALUE OF FUNCTION
260  F0=F0+A(I)*S
269  REM - CALCULATE VALUE OF DERIVATIVE
270  F1=F1+B(I)*S
280  S=S*X
290  NEXT I
299  REM - TEST FOR A ZERO DERIVATIVE; IF YES, STOP SEARCH, PRINT
300  IF F1=0  THEN  360
309  REM - GET NEW GUESS USING PREVIOUS GUESS
310  S=X-F0/F1
319  REM - IF NEW GUESS = LAST GUESS THEN STOP SEARCH, PRINT
320  IF X=S  THEN  360
329  REM - SAVE LAST GUESS
330  X=S
340  IF Q>100  THEN  490
350  GOTO  210
360  PRINT "DERIVATIVE = 0 AT X =";X
370  GOTO  180
380  PRINT
390  PRINT " ROOT","  ERROR","  DERIVATIVE"
400  PRINT X,F0,F1
410  PRINT
419  REM - RERUN TO FIND ANOTHER ROOT IN SAME FUNCTION?
420  PRINT "ANOTHER VALUE (1=YES, 0=NO)";
430  INPUT A
440  IF A=1  THEN  170
449  REM - RESTART OR END PROGRAM?
450  PRINT "ANOTHER FUNCTION (1=YES, 0=NO)";
460  INPUT A
470  IF A=1  THEN  30
480  GOTO  550
489  REM - PRINT CALCULATED VALUES AFTER 100 ITERATIONS; SEARCH 100 MORE
490  PRINT "100 ITERATIONS COMPLETED:"
500  PRINT "  X =";X;"  F(X) =";F0
510  PRINT "   CONTINUE (1=YES, 0=NO)";
520  INPUT A
530  IF A=1  THEN  200
540  GOTO  420
550  END
```

Roots of Polynomials: Half-interval Search

This program calculates roots of polynomials within a given interval. The program first conducts a random search within the given interval for two points with opposite signs. If a change of sign is found, then the root is calculated by the half-interval search method. If there is no change of sign found, another interval will be asked for.

Errors may result in this program for a couple of reasons. First, a root may be calculated when it should not be. This may happen if the lowest point is so close to zero that a root is found due to round-off error. Second, two roots may be so close together that the program never finds the opposite signs between them. The result in this case is that neither root is calculated.

It is necessary to enter the equation before you run the program. The equation will be defined as a function of x at statement 30. For example, if you want to find roots of the function $f(x) = 4x^4-2.5x^2-x+.5$, you will enter:

```
30 DEFFNR(X)=4*X↑4-2.5*X↑2-X+.5
```

Example:

Find a root of the function $f(x) = 4x^4-2.5x^2-x+.5$.

```
:30 DEFFNR(X)=4*X↑4-2.5*X↑2-X+.5
:RUN
ROOTS OF POLYNOMIALS: HALF-INTERVAL SEARCH

(TO END SEARCH ENTER 0,0)
INTERVAL (LOWER, UPPER LIMIT)? -1,0
NO CHANGE OF SIGN FOUND
INTERVAL (LOWER, UPPER LIMIT)? 0,1
ROOT = .3035792010268

INTERVAL (LOWER, UPPER LIMIT)? 0,0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "ROOTS OF POLYNOMIALS: HALF-INTERVAL SEARCH"
 20  PRINT
 30  REM - ENTER FUNCTION (DEFFNR(X)="FUNCTION") HERE
 40  DIM D(3)
 50  PRINT "(TO END SEARCH ENTER 0,0)"
 59  REM - ESTABLISH INTERVAL OF RANDOM SEARCH
 60  PRINT "INTERVAL (LOWER, UPPER LIMIT)";
 70  INPUT A,B
 79  REM - TEST FOR USABLE LIMITS ENTERED
 80  IF A<>B  THEN  120
 89  REM - END PROGRAM?
 90  IF A=0  THEN  430
100  PRINT "--INTERVAL LIMITS CANNOT BE EQUAL--"
```

```
110  GOTO   60
120  IF A<B  THEN  150
130  PRINT "--LOWER LIMIT MUST BE ENTERED FIRST--"
140  GOTO   60
150  A1=SGN(FNR(A))
160  B1=SGN(FNR(B))
169  REM - TEST FOR ROOT AT EITHER LIMIT
170  IF A1*B1=0  THEN  360
179  REM - TEST FOR OPPOSITE SIGNS AT INTERVAL LIMITS
180  IF A1*B1<0  THEN  280
189  REM - LOOP TO SEARCH 1000 NUMBERS FOR OPPOSITE SIGNS IN FUNCTION
190  FOR I=1  TO 1000
200  X=A+RND(2)*(B-A)
210  X1=SGN(FNR(X))
219  REM - TEST FOR ROOT AT RANDOM NUMBER; IF YES, END SEARCH, PRINT
220  IF X1=0  THEN  400
229  REM - TEST FOR OPPOSITE SIGNS AT RANDOM NUMBER AND LOWER LIMIT
230  IF A1*X1<0  THEN  270
239  REM - TRY ANOTHER RANDOM NUMBER
240  NEXT I
250  PRINT "NO CHANGE OF SIGN FOUND"
260  GOTO   60
269  REM - CHANGE OF SIGN FOUND; CALCULATE ROOT
270  B=X
278  REM - STORE POSITIVE POINT IN D(3), NEGATIVE POINT IN D(1)
279  REM - D(1) AND D(3) BECOME INTERVAL LIMITS
280  D(2+A1)=A
290  D(2-A1)=B
295  REM - CALCULATE MIDPOINT BETWEEN THE TWO LIMITS
300  Y=(D(1)+D(3))/2
310  Y1=SGN(FNR(Y))
319  REM - TEST FOR ROOT AT MIDPOINT
320  IF Y1=0  THEN  400
329  REM - GET A NEW LIMIT TO CLOSE IN ON ROOT
330  D(2+Y1)=Y
339  REM - TEST FOR A VALUE CLOSE ENOUGH TO ZERO TO ASSUME A ROOT
340  IF ABS(D(1)-D(3))/ABS(D(1)+ABS(D(3)))<5E-6  THEN  400
349  REM - RETEST WITH NEW LIMITS
350  GOTO  300
359  REM - ROOT AT AN INTERVAL LIMIT; FIND WHICH LIMIT, PRINT
360  IF A1=0  THEN  390
370  Y=B
380  GOTO  400
390  Y=A
400  PRINT "ROOT =";Y
410  PRINT
419  REM - RESTART PROGRAM
420  GOTO   60
430  END
```

Trig Polynomial

This program solves a trigonometric function for a given angle. The function must be in the following form:

$$f(x) = A_1 \sin(x) + B_1 \cos(x) + A_2 \sin(2x) + B_2 \sin(2x) \ldots + A_n \sin(n \cdot x) + B_n \cos(n \cdot x)$$

where n = the number of pairs of coefficients.

The coefficients of the function are to be entered in a data statement at line 30. The data statement will include the number of pairs of coefficients (n) and the coefficients of the polynomial. It will be entered as follows:

$$30 \text{ DATA } n, A_1, B_1, A_2, B_2, \ldots A_n, B_n$$

mple:

Solve the following equation when the angle equals 45°, 90° and 105°:

$$f(x) = \sin(x) + 2 \cdot \cos(x) - 2 \cdot \sin(2x) + \cos(2x) + 5 \cdot \sin(3x) - 3 \cdot \cos(3x)$$

```
 DATA 3,1,2,-2,1,5,-3
N
 POLYNOMIAL

TER ANGLE=99999 TO END PROGRAM)
GLE? 45
 45 )= 3.095587494888

GLE? 90
 90 )=-2.831680950826

GLE? 105
 105 )=-1.546848370549

GLE? 99999

 PROGRAM
```

GRAM LISTING

```
 10  PRINT "TRIG POLYNOMIAL"
 20  PRINT
 30  REM - ENTER NUMBER OF PAIRS OF TERMS AND COEFFICIENTS HERE
 40  PRINT "(ENTER ANGLE=99999 TO END PROGRAM)"
 50  PRINT "ANGLE";
 60  INPUT R
 69  REM - END PROGRAM?
 70  IF R=99999  THEN  180
 79  REM - GET NUMBER OF PAIRS OF TERMS IN POLYNOMIAL
 80  READ N
 89  REM - LOOP TO GET VALUES OF COEFFICIENTS FROM DATA TABLE
 90  FOR I=1  TO N
100  READ A,B
```

```
109  REM - CALCULATE VALUE OF FUNCTION AT ANGLE X
110  Z=Z+A*SIN(I*R)+B*COS(I*R)
120  NEXT I
129  REM - PRINT RESULTS
130  PRINT "F(";R;")=";Z
139  REM - PREPARE TO REREAD FUNCTION COEFFICIENTS
140  RESTORE
150  PRINT
160  Z=0
169  REM - RESTART PROGRAM
170  GOTO   50
180  END
```

Simultaneous Equations

This program solves a system of linear equations. The number of unknown coefficients in each equation must equal the number of equations being solved. You must enter the coefficients of each equation.

The dimension statement at line 30 limits the number of equations which may be solved. You may change this limit according to the following scheme:

```
30 DIM A(R,R+1)
```

where R = the maximum number of equations.

Example:

Solve the following system of equations:

$$\begin{aligned} x + 2x + 3x &= 4 \\ 3x + 6x &= 1 \\ -3x + 4x - 2x &= 0 \end{aligned}$$

```
:30 DIM A(3,4)
:RUN
SIMULTANEOUS EQUATIONS

NUMBER OF EQUATIONS? 3
COEFFICIENT MATRIX:
EQUATION 1
  COEFFICIENT 1 ? 1
  COEFFICIENT 2 ? 2
  COEFFICIENT 3 ? 3
  CONSTANT? 4
EQUATION 2
  COEFFICIENT 1 ? 3
  COEFFICIENT 2 ? 6
  COEFFICIENT 3 ? 0
  CONSTANT? 1
EQUATION 3
  COEFFICIENT 1 ? -3
  COEFFICIENT 2 ? 4
  COEFFICIENT 3 ? -2
  CONSTANT? 0

X 1 =-.356
X 2 = .344
X 3 = 1.222

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "SIMULTANEOUS EQUATIONS"
 20  PRINT
 29  REM - LIMIT A() TO A(R,R+1)  WHERE R=MAX. NO. OF EQUATIONS
 30  DIM A(9,10)
 40  PRINT "NUMBER OF EQUATIONS";
 50  INPUT R
 60  PRINT "COEFFICIENT MATRIX:"
 70  FOR J=1  TO R
 80  PRINT "EQUATION";J
 90  FOR I=1  TO R+1
100  IF I=R+1  THEN  130
110  PRINT "  COEFFICIENT";I;
120  GOTO  140
130  PRINT "  CONSTANT";
140  INPUT A(J,I)
150  NEXT I
160  NEXT J
170  FOR J=1  TO R
178  REM - STATEMENTS 180 TO 220 FIND THE FIRST EQUATION WITH A
179  REM - NON-ZERO COEFFICIENT FOR THE CURRENT COLUMN
180  FOR I=J  TO R
190  IF A(I,J)<>0  THEN  230
200  NEXT I
210  PRINT "NO UNIQUE SOLUTION"
220  GOTO  440
229  REM - STATEMENTS 230 TO 270 MOVE THAT EQUATION UP TO THE CURRENT
230  FOR K=1  TO R+1
240  X=A(J,K)
250  A(J,K)=A(I,K)
260  A(I,K)=X
270  NEXT K
279  REM - STATEMENTS 280 TO 310 GET A 1 COEFFICIENT IN THE FIRST NON
   0 COLUMN OF THE CURRENT ROW
280  Y=1/A(J,J)
290  FOR K=1  TO R+1
300  A(J,K)=Y*A(J,K)
310  NEXT K
318  REM - STATEMENTS 320 TO 380 SUBTRACT THE CURRENT EQUATION FROM
319  REM - THE OTHER ROWS
320  FOR I=1  TO R
330  IF I=J  THEN  380
340  Y=-A(I,J)
350  FOR K=1  TO R+1
360  A(I,K)=A(I,K)+Y*A(J,K)
370  NEXT K
380  NEXT I
389  REM - THIS PROCESS IS REPEATED FOR ALL EQUATIONS
390  NEXT J
400  PRINT
409  REM - PRINT SOLUTIONS
410  FOR I=1  TO R
420  PRINT "X";I;"=";INT(A(I,R+1)*1000+.5)/1000
430  NEXT I
440  END
```

Linear Programming

Courtesy: Harold Hanes
Earlham College
Richmond, Indiana

This program uses the simplex method to solve a linear programming problem. You must provide the coefficients of the objective function and the coefficients, relation and constant of each constraint. This information is entered in DATA statements before you run the program.

After you load the program, enter the DATA statements according to the following instructions. If you run more than one problem, remember to clear out all DATA statements from the previous problem before running the new problem. Our DATA statements begin at line 3000.

1) Arrange your problem constraints according to their relation, so that the "less than" inequalities precede the equalities, which in turn precede the "greater than" inequalities.
2) Type in as DATA the coefficients of the constraints, in the order the constraints were arranged in step 1. Do not include coefficients for slack, surplus, or artificial variables. Do include a '0' coefficient for any variable that doesn't appear in a particular constraint.
3) Type in as DATA the constants of the constraints (right-hand sides of the constraints) in the same order as you entered the rows of coefficients. These values cannot be negative.
4) Type in as DATA the coefficients of the objective function.

You must select whether the problem solution is to be a minimum or maximum value. The program also asks you to enter the total number of constraints and the number of variables to allow for each, and the number of "less than", "equal" and "greater than" constraints you are considering.

The dimension statement at line 20 limits the number of variables and constraints you may enter. You can change these limits according to the following scheme:

```
20 DIM A(C+2,V+C+G+1),B(C+2)
```

where: C = number of constraints
V = number of variables
G = number of "greater than" constraints

Example:

A manufacturer wishes to produce 100 pounds of an alloy which is 83% lead, 14% iron, and 3% antimony. He has available five alloys with the following compositions and prices:

	alloy 1	alloy 2	alloy 3	alloy 4	alloy 5
	90	80	95	70	30
	5	5	2	30	70
	5	15	3	0	0
	$6.13	$7.12	$5.85	$4.57	$3.96

How should he combine these alloys to get the desired product at minimum cost?

Note that this problem results in the following system of equations:

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 + x_5 &= 100 \\
.90x_1 + .80x_2 + .95x_3 + .70x_4 + .30x_5 &= 83 \\
.05x_1 + .05x_2 + .02x_3 + .30x_4 + .70x_5 &= 14 \\
.05x_1 + .15x_2 + .03x_3 + \quad + \quad &= 3 \\
6.13x_1 + 7.12x_2 + 5.85x_3 + 4.57x_4 + 3.96x_5 &= Z \text{ (min)}
\end{aligned}
$$

```
:RUN
LINEAR PROGRAMMING


TYPE '1' FOR MAXIMIZATION, OR '-1' FOR MINIMIZATION? -1
TYPE NUMBER OF CONSTRAINTS; NUMBER OF VARIABLES? 4,5
NUMBER OF LESS THAN, EQUAL, GREATER CONSTRAINTS? 0,4,0


YOUR VARIABLES  1 THROUGH 5
ARTIFICIAL VARIABLES 6 THROUGH 9



ANSWERS:
PRIMAL VARIABLES:
VARIABLES          VALUE
 2                  10.4347826087
 3                  47.82608695654
 4                  41.73913043478
DUAL VARIABLES:
VARIABLE           VALUE
VALUE OF OBJECTIVE FUNCTION 544.8260869565



END PROGRAM


PROGRAM LISTING

  10  PRINT "LINEAR PROGRAMMING"
  15  PRINT
  19  REM - LINEAR PROGRAMMING, SIMPLEX METHOD
  20  DIM A(6,10),B(6)
  30  PRINT
  40  PRINT "TYPE '1' FOR MAXIMIZATION, OR '-1' FOR MINIMIZATION";
  50  INPUT Z
  60  Z=-Z
  70  PRINT "TYPE NUMBER OF CONSTRAINTS, NUMBER OF VARIABLES";
  80  INPUT M,N
  90  PRINT "NUMBER OF LESS THAN, EQUAL, GREATER CONSTRAINTS";
 100  INPUT L,E,G
 110  IF M=L+E+G  THEN  140
 120  PRINT "DATA ON CONSTRAINTS INCONSISTENT, TRY AGAIN."
 130  GOTO   90
 139  REM - THIS IS THE INITIALIZATION ROUTINE
 140  C=N+M+G
 150  C1=C+1
 160  C2=N+L+G
 170  M1=M+1
 180  M2=M+2
 190  PRINT
 200  FOR I=1  TO M2
 210  FOR J=1  TO C1
 220  A(I,J)=0
 230  NEXT J
```

```
240  NEXT I
250  FOR I=1  TO M
260  B(I)=0
270  NEXT I
280  FOR I=1  TO M
290  FOR J=1  TO N
300  READ A(I,J)
310  IF I<=L  THEN  330
320  A(M1,J)=A(M1,J)-A(I,J)
330  NEXT J
340  IF I>L  THEN  380
350  B(I)=N+I
360  A(I,N+I)=1
370  GOTO  440
380  B(I)=N+G+I
390  A(I,N+G+I)=1
400  IF I>L+E  THEN  420
410  GOTO  440
420  A(I,N+I-E)=-1
430  A(M1,N+I-E)=1
440  NEXT I
450  FOR I=1  TO M
460  READ A(I,C1)
470  NEXT I
480  FOR J=1  TO N
490  READ A(M2,J)
500  A(M2,J)=Z*A(M2,J)
510  NEXT J
520  PRINT
540  PRINT "YOUR VARIABLES 1 THROUGH";N
550  IF L=0  THEN  570
560  PRINT "SLACK VARIABLES";N+1;"THROUGH";N+L
570  IF G=0  THEN  590
580  PRINT "SURPLUS VARIABLES";N+L+1;"THROUGH";C
590  IF L=M  THEN  780
600  PRINT "ARTIFICIAL VARIABLES";C2+1;"THROUGH";C
610  M3=M1
620  GOSUB 1040
630  PRINT
640  FOR I1=1  TO M
650  IF B(I1)<=C2  THEN  760
660  IF A(I1,C1)<=.00001  THEN  690
670  PRINT "THE PROBLEM HAS NO FEASIBLE SOLUTION."
680  GOTO 3060
690  FOR J1=1  TO C2
700  IF ABS(A(I1,J1))<=.00001  THEN  750
710  R=I1
720  S=J1
730  GOSUB 1260
740  J1=C2
750  NEXT J1
760  NEXT I1
780  PRINT
790  M3=M2
800  GOSUB 1040
```

```
 830  PRINT
 840  PRINT "ANSWERS:"
 850  PRINT "PRIMAL VARIABLES:"
 860  PRINT "VARIABLES","VALUE"
 870  FOR J=1  TO C2
 880  FOR I=1  TO M
 890  IF B(I)<>J  THEN  920
 900  PRINT J,A(I,C1)
 910  I=M
 920  NEXT I
 930  NEXT J
 940  PRINT "DUAL VARIABLES:"
 950  PRINT "VARIABLE","VALUE"
 960  IF L=0  THEN 1000
 970  FOR I=1  TO L
 980  PRINT I,-Z*A(M2,N+I)
 990  NEXT I
1000  PRINT "VALUE OF OBJECTIVE FUNCTION";-Z*A(M2,C1)
1010  PRINT
1020  PRINT
1030  GOTO 3060
1038  REM - OPTIMIZATION ROUTINE
1039  REM - FIRST PRICE OUT COLUMNS
1040  P=-.00001
1050  FOR J=1  TO C2
1060  IF A(M3,J)>=P  THEN 1090
1070  S=J
1080  P=A(M3,J)
1090  NEXT J
1100  IF P=-.00001  THEN 1440
1110  GOSUB 1130
1120  GOSUB 1210
1125  GOTO 1040
1129  REM - NOW FIND WHICH VARIABLE LEAVE BASIS
1130  Q=1.E+38
1140  FOR I=1  TO M
1150  IF A(I,S)<=.00001  THEN 1190
1160  IF A(I,C1)/A(I,S)>=Q  THEN 1190
1170  R=I
1180  Q=A(I,C1)/A(I,S)
1190  NEXT I
1200  RETURN
1210  IF Q=1.E+38  THEN 1240
1220  GOSUB 1260
1230  RETURN
1240  PRINT "THE SOLUTION IS UNBOUNDED."
1250  GOTO 3060
1259  REM - PERFORM PIVOTING
1260  P=A(R,S)
1270  FOR I=1  TO M2
1280  IF I=R  THEN 1350
1290  FOR J=1  TO C1
1300  IF J=S  THEN 1340
1310  A(I,J)=A(I,J)-A(I,S)*A(R,J)/P
1320  IF ABS(A(I,J))>=.00001  THEN 1340
1330  A(I,J)=0
```

```
1340  NEXT J
1350  NEXT I
1360  FOR J=1  TO C1
1370  A(R,J)=A(R,J)/P
1380  NEXT J
1390  FOR I=1  TO M2
1400  A(I,S)=0
1410  NEXT I
1420  A(R,S)=1
1430  B(R)=S
1440  RETURN
2996  REM - *** DO THE FOLLOWING STEPS BEFORE RUNNING THE PROGRAM ***
2997  REM - TYPE IN COEFFICIENTS OF '<','=', AND '>' CONSTRAINTS IN
   DATA STATEMENTS STARTING AT LINE 3000, A SEPARATE DATA STATEMENT
   FOR EACH CONSTRAINT (LINES 3000 - 3030 IN OUR EXAMPLE)
2998  REM - TYPE IN CONSTANTS OF THE CONSTRAINTS IN A DATA STATEMENT
   FOLLOWING THE COEFFICIENT DATA, AND IN THE SAME ORDER AS THE CON
   STRAINT DATA WERE ENTERED (LINE 3040 IN OUR EXAMPLE)
2999  REM - TYPE IN THE COEFFICIENTS OF THE OBJECTIVE FUNCTION IN A
   DATA STATEMENT (LINE 3050 IN OUR EXAMPLE) FOLLOWING THE CONSTANTS
   DATA
3000  DATA 1,1,1,1,1
3010  DATA .9,.8,.95,.7,.3
3020  DATA .05,.05,.02,.3,.7
3030  DATA .05,.15,.03,0,0
3040  DATA 100,83,14,3
3050  DATA 6.13,7.12,5.85,4.57,3.96
3060  END
```

Matrix Addition, Subtraction, Scalar Multiplication

This program adds or subtracts two matrices, or multiplies a matrix by a given scalar. You must input the value of each element of each matrix. To perform addition or subtraction the dimensions of the two matrices must be equal.

The dimension of the matrices may be increased or decreased depending on the amount of memory available in your system. Statement 30 may be changed to:

```
30 DIM A(X,Y), B(X,Y)
```

where (*X,Y*) is your limit on the dimension of the matrices.

Example:

Find the sum of the following matrices, then multiply the resultant matrix by 3.

$$\begin{bmatrix} 1 & 0 & -1 \\ 5 & 8 & .5 \\ -1 & 2 & 0 \end{bmatrix} \qquad \begin{bmatrix} -5 & -1 & 2 \\ 6 & -.1 & 0 \\ 3 & 4 & -2 \end{bmatrix}$$

```
:RUN
MATRIX ADDITION, SUBTRACTION, SCALAR MULTIPLICATION

1=ADDITION
2=SUBTRACTION
3=SCALAR MULTIPLICATION
WHICH OPERATION? 1
DIMENSION OF MATRIX (R,C)? 3,3
MATRIX 1:
ROW 1
VALUE COLUMN 1 ? 1
VALUE COLUMN 2 ? 0
VALUE COLUMN 3 ? -1
ROW 2
VALUE COLUMN 1 ? 5
VALUE COLUMN 2 ? 8
VALUE COLUMN 3 ? .5
ROW 3
VALUE COLUMN 1 ? -1
VALUE COLUMN 2 ? 2
VALUE COLUMN 3 ? 0
MATRIX 2:
ROW 1
VALUE COLUMN 1 ? -5
VALUE COLUMN 2 ? -1
VALUE COLUMN 3 ? 2
ROW 2
VALUE COLUMN 1 ? 6
VALUE COLUMN 2 ? -.1
VALUE COLUMN 3 ? 0
ROW 3
VALUE COLUMN 1 ? 3
```

```
VALUE COLUMN 2 ? 4
VALUE COLUMN 3 ? -2
-4    -1     1
 11     7.9     .5
 2     6    -2

MORE DATA? (1=YES,0=NO)? 1
WHICH OPERATION? 3
VALUE OF SCALAR? 3
DIMENSION OF MATRIX (R,C)? 3,3
MATRIX 1:
ROW 1
VALUE COLUMN 1 ? -4
VALUE COLUMN 2 ? -1
VALUE COLUMN 3 ? 1
ROW 2
VALUE COLUMN 1 ? 11
VALUE COLUMN 2 ? 7.9
VALUE COLUMN 3 ? .5
ROW 3
VALUE COLUMN 1 ? 2
VALUE COLUMN 2 ? 6
VALUE COLUMN 3 ? -2
-12    -3     3
 33     23.7     1.5
 6     18    -6

MORE DATA? (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "MATRIX ADDITION, SUBTRACTION, SCALAR MULTIPLICATION"
 20  PRINT
 29  REM - ARRAYS SHOULD BE SET TO DIMENSIONS OF MATRICES
 30  DIM A(3,3), B(3,3)
 40  PRINT "1=ADDITION"
 50  PRINT "2=SUBTRACTION"
 60  PRINT "3=SCALAR MULTIPLICATION"
 69  REM - SELECT OPERATION BY ENTERING THE NUMBER (1-3) OF THE OPERATION
 70  PRINT "WHICH OPERATION";
 80  INPUT D
 89  REM - TEST FOR ADDITION OR SUBRACTION
 90  IF D<>3  THEN  120
100  PRINT "VALUE OF SCALAR";
110  INPUT S
120  PRINT "DIMENSION OF MATRIX (R,C)";
130  INPUT R,C
138  REM - LOOP TO ENTER MATRIX VALUES
139  REM - FOR SUBTRACTION, MATRIX 2 SUBTRACTED FROM MATRIX 1
140  FOR K=1  TO 2
150  IF K=2  THEN  180
160  PRINT "MATRIX 1:"
170  GOTO  190
180  PRINT "MATRIX 2:"
```

```
190  FOR J=1  TO R
200  PRINT "ROW";J
210  FOR I=1  TO C
220  PRINT "VALUE COLUMN";I;
230  IF K=2  THEN  260
240  INPUT A(J,I)
250  GOTO  270
260  INPUT B(J,I)
270  NEXT I
280  NEXT J
289  REM - ONLY ONE MATRIX USED FOR SCALAR MULTIPLICATION
290  IF D=3  THEN  310
300  NEXT K
309  REM - STATEMENTS 310 TO 410 PERFORM REQUESTED OPERATION AND PRINT RES
   ULTANT MATRIX
310  FOR J=1  TO R
320  FOR I=1  TO C
330  IF D<>2  THEN  350
340  B(J,I)=-B(J,I)
350  IF D=3  THEN  380
360  PRINT A(J,I)+B(J,I);"   ";
370  GOTO  390
380  PRINT A(J,I)*S;"   ";
390  NEXT I
399  REM - ADVANCE OUTPUT DEVICE TO PRINT NEXT ROW
400  PRINT
410  NEXT J
420  PRINT
429  REM - RESTART OR END PROGRAM?
430  PRINT "MORE DATA? (1=YES,0=NO)";
440  INPUT D
450  IF D=1  THEN   70
460  END
```

Matrix Multiplication

This program multiplies two matrices. The first matrix is multiplied by the second. You must input the elements of each matrix.

In order for this operation to be performed the number of rows in the first matrix must equal the number of columns in the second matrix.

The dimensions of the matrices are presently limited to 10 x 10. This limit may be increased or decreased by altering line 30 according to the following scheme:

```
30 DIM A(X,Y), B(Z,X)
```

where: (X,Y) = dimension of matrix 1
(Z,X) = dimension of matrix 2

Example:

Multiply matrix 1 by matrix 2.

$$1\begin{pmatrix} 2 & -1 & 4 & 1 & 2 \\ 1 & 0 & 1 & 2 & -1 \\ 2 & 3 & -1 & 0 & -2 \end{pmatrix}$$

$$2\begin{pmatrix} -2 & -1 & 2 \\ 0 & 2 & 1 \\ -1 & 1 & 4 \\ 3 & 0 & -1 \\ 2 & 1 & 2 \end{pmatrix}$$

```
:30 DIM A(3,5), B(5,3)
:RUN
MATRIX MULTIPLICATION

DIMENSION OF MATRIX 1 (R,C)? 3,5
DIMENSION OF MATRIX 2 (R,C)? 5,3
MATRIX 1:
ROW 1
VALUE COLUMN 1 ? 2
VALUE COLUMN 2 ? -1
VALUE COLUMN 3 ? 4
VALUE COLUMN 4 ? 1
VALUE COLUMN 5 ? 2
ROW 2
VALUE COLUMN 1 ? 1
VALUE COLUMN 2 ? 0
VALUE COLUMN 3 ? 1
VALUE COLUMN 4 ? 2
VALUE COLUMN 5 ? -1
ROW 3
VALUE COLUMN 1 ? 2
VALUE COLUMN 2 ? 3
VALUE COLUMN 3 ? -1
VALUE COLUMN 4 ? 0
```

```
VALUE COLUMN 5 ? -2

MATRIX 2:
ROW 1
VALUE COLUMN 1 ? -2
VALUE COLUMN 2 ? -1
VALUE COLUMN 3 ? 2
ROW 2
VALUE COLUMN 1 ? 0
VALUE COLUMN 2 ? 2
VALUE COLUMN 3 ? 1
ROW 3
VALUE COLUMN 1 ? -1
VALUE COLUMN 2 ? 1
VALUE COLUMN 3 ? 4
ROW 4
VALUE COLUMN 1 ? 3
VALUE COLUMN 2 ? 0
VALUE COLUMN 3 ? -1
ROW 5
VALUE COLUMN 1 ? 2
VALUE COLUMN 2 ? 1
VALUE COLUMN 3 ? 2

-1     2     22
 1    -1      2
-7     1     -1

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "MATRIX MULTIPLICATION"
 20  PRINT
 29  REM - ARRAYS A AND B SHOULD BE SET TO DIMENSIONS OF MATRICES
 30  DIM A(10,10), B(10,10)
 40  PRINT "DIMENSION OF MATRIX 1 (R,C)";
 50  INPUT R1,C1
 60  PRINT "DIMENSION OF MATRIX 2 (R,C)";
 70  INPUT R2,C2
 79  REM - # OF COLUMNS IN MATRIX 1 MUST EQUAL # OF ROWS IN MATRIX 2
 80  IF C1=R2  THEN  110
 90  PRINT "CANNOT BE MULTIPLIED; OTHER DIMENSIONS NECESSARY"
100  GOTO   40
109  REM - ENTER MATRIX VALUES
110  PRINT "MATRIX 1:"
120  FOR J=1  TO R1
130  PRINT "ROW";J
140  FOR I=1  TO C1
150  PRINT "VALUE COLUMN";I;
160  INPUT A(J,I)
170  NEXT I
180  NEXT J
190  PRINT
200  PRINT "MATRIX 2:"
210  FOR J=1  TO R2
```

```
220  PRINT "ROW";J
230  FOR I=1  TO C2
240  PRINT "VALUE COLUMN";I;
250  INPUT B(J,I)
260  NEXT I
270  NEXT J
280  PRINT
289  REM - PERFORM MATRIX MULTIPLICATION, PRINT RESULTANT MATRIX
290  FOR I=1  TO R1
300  FOR J=1  TO C2
310  S=0
320  FOR K=1  TO C1
330  S=S+A(I,K)*B(K,J)
340  NEXT K
350  PRINT S;"  ";
360  NEXT J
369  REM - ADVANCE OUTPUT DEVICE TO PRINT NEXT ROW
370  PRINT
380  NEXT I
390  END
```

Matrix Inversion

This program inverts a square matrix. The inversion is performed by a modified Gauss-Jordan elimination method.

The dimensions of the matrix are presently limited to 10 x 10. This limit may be increased or decreased by altering line 30 according to the following scheme:

```
30 DIM A(R,R), B(R,R)
```

where R = number of rows (or columns) in the matrix.

Example:

Invert matrix A.

$$A \begin{Bmatrix} 3 & 5 & -1 & -4 \\ 1 & 4 & -.7 & -3 \\ 0 & -2 & 0 & 1 \\ -2 & 6 & 0 & .3 \end{Bmatrix}$$

```
:RUN
MATRIX INVERSION

DIMENSION OF MATRIX? 4
MATRIX ELEMENTS:
ROW 1
VALUE COLUMN 1 ? 3
VALUE COLUMN 2 ? 5
VALUE COLUMN 3 ? -1
VALUE COLUMN 4 ? -4
ROW 2
VALUE COLUMN 1 ? 1
VALUE COLUMN 2 ? 4
VALUE COLUMN 3 ? -.7
VALUE COLUMN 4 ? -3
ROW 3
VALUE COLUMN 1 ? 0
VALUE COLUMN 2 ? -2
VALUE COLUMN 3 ? 0
VALUE COLUMN 4 ? 1
ROW 4
VALUE COLUMN 1 ? -2
VALUE COLUMN 2 ? 6
VALUE COLUMN 3 ? 0
VALUE COLUMN 4 ? .3

 .654   -.935   -.191    1.40000000E-02
 .198   -.283   -.103     .156
 .368   -1.955  -4.263    -.425
 .397   -.567    .793     .312

END PROGRAM
```

```
GRAM LISTING
 0  PRINT "MATRIX INVERSION"
 0  PRINT
 9  REM - A() AND B() SHOULD BOTH BE SET TO THE DIMENSIONS OF THE MAT
  RIX
 0  DIM A(10,10), B(10,10)
 9  REM - MATRIX IS SQUARE SO ONLY ONE DIMENSION IS NEEDED
 0  PRINT "DIMENSION OF MATRIX";
 0  INPUT R
 0  PRINT "MATRIX ELEMENTS:"
 9  REM - ENTER MATRIX ELEMENTS
 0  FOR J=1  TO R
 0  PRINT "ROW";J
 0  FOR I=1  TO R
 0  PRINT "VALUE COLUMN";I;
 0  INPUT A(J,I)
 0  NEXT I
 0  B(J,J)=1
 0  NEXT J
 9  REM - STATEMENTS 150 TO 420 INVERT MATRIX
 0  FOR J=1  TO R
 0  FOR I=J  TO R
 0  IF A(I,J)<>0  THEN  210
 0  NEXT I
 0  PRINT "SINGULAR MATRIX"
 0  GOTO  500
 0  FOR K=1  TO R
 0  S=A(J,K)
 0  A(J,K)=A(I,K)
 0  A(I,K)=S
 0  S=B(J,K)
 0  B(J,K)=B(I,K)
 0  B(I,K)=S
 0  NEXT K
 0  T=1/A(J,J)
 0  FOR K=1  TO R
 0  A(J,K)=T*A(J,K)
 0  B(J,K)=T*B(J,K)
 0  NEXT K
 0  FOR L=1  TO R
 0  IF L=J  THEN  410
 0  T=-A(L,J)
 0  FOR K=1  TO R
 0  A(L,K)=A(L,K)+T*A(J,K)
 0  B(L,K)=B(L,K)+T*B(J,K)
 0  NEXT K
 0  NEXT L
 0  NEXT J
 0  PRINT
 9  REM - PRINT RESULTANT MATRIX
 0  FOR I=1  TO R
 0  FOR J=1  TO R
 9  REM - ROUND OFF, PRINT
 0  PRINT INT(B(I,J)*1000+.5)/1000;" ";
 0  NEXT J
 9  REM - ADVANCE OUTPUT DEVICE TO PRINT NEXT LINE
 0  PRINT
 0  NEXT I
 0  END
```

Permutations and Combinations

This program computes the number of permutations and combinations of *N* objects taken *D* at a time.

Examples:

How many permutations and combinations can be made of the 26 letters of the alphabet, taking five at a time?

How many different ways can 12 people sit on a bench if there is only room for two at a time?

```
:RUN
PERMUTATIONS AND COMBINATIONS

(ENTER 0 TO END PROGRAM)
TOTAL NUMBER OF OBJECTS? 26
SIZE OF SUBGROUP? 5
 7893600 PERMUTATIONS
 65780 COMBINATONS

TOTAL NUMBER OF OBJECTS? 12
SIZE OF SUBGROUP? 2
 132 PERMUTATIONS
 66 COMBINATONS

TOTAL NUMBER OF OBJECTS? 0

END PROGRAM
```

PROGRAM LISTING

```
  10  PRINT "PERMUTATIONS AND COMBINATIONS"
  20  PRINT
  30  PRINT "(ENTER 0 TO END PROGRAM)"
  40  PRINT "TOTAL NUMBER OF OBJECTS";
  50  INPUT N
  59  REM - TEST FOR END OF PROGRAM
  60  IF N=0  THEN  280
  70  PRINT "SIZE OF SUBGROUP";
  80  INPUT D
  89  REM - SIZE OF SUBGROUP CANNOT BE LARGER THAN SIZE OF GROUP
  90  IF D<=N  THEN  130
 100  PRINT "SUBGROUP TOO LARGE"
 110  PRINT
 120  GOTO  40
 129  REM - LINES 130 TO 200 COMPUTE PERMUTATIONS
 130  P=1
 140  C=1
 150  FOR I=N-D+1  TO N
 159  REM - DON'T ALLOW NUMBER SIZE TO OVERFLOW MACHINE CAPACITY
 160  IF 9.9E62/I>=P  THEN  190
 170  PRINT "MORE THAN 9.9E62 PERMUTATIONS"
```

```
180  GOTO  280
190  P=P*I
200  NEXT I
209  REM - COMPUTE INTERMEDIATE FACTORIAL FOR COMBINATIONS
210  FOR J=2  TO D
220  C=C*J
230  NEXT J
240  PRINT P;"PERMUTATIONS"
250  PRINT P/C;"COMBINATONS"
260  PRINT
269  REM - RESTART PROGRAM
270  GOTO   40
280  END
```

Mann-Whitney *U* Test

This program performs the Mann-Whitney *U* test on samples from two populations.

The dimension statement on line 30 limits the size of the samples. You can increase or decrease the dimension limits according to the following scheme:

```
30 DIM X(M), Y(N)
```

where: *M* = maximum size of first sample
N = maximum size of second sample

Example:

A group of ten women and a group of ten men were asked to rate the flavor of a frozen T.V. dinner on a scale of one to ten. The table below lists the scores. Count the number of times the women's scores are lower than the men's, and vice-versa.

women	1	3	4	3	6	8	9	7	8	4
men	7	9	8	5	10	9	10	6	5	2

```
:30 DIM X(10),Y(10)
:RUN
MANN-WHITNEY U-TEST

SAMPLE 1 :
  SIZE? 10
   DATA 1 ? 1
   DATA 2 ? 3
   DATA 3 ? 4
   DATA 4 ? 3
   DATA 5 ? 6
   DATA 6 ? 8
   DATA 7 ? 9
   DATA 8 ? 7
   DATA 9 ? 8
   DATA 10 ? 4

SAMPLE 2 :
  SIZE? 10
   DATA 1 ? 7
   DATA 2 ? 9
   DATA 3 ? 6
   DATA 4 ? 5
   DATA 5 ? 10
   DATA 6 ? 9
   DATA 7 ? 10
   DATA 8 ? 6
   DATA 9 ? 5
   DATA 10 ? 2

FIRST SAMPLE PRECEDING, U = 70
SECOND SAMPLE PRECEDING, U = 30

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "MANN-WHITNEY U-TEST"
 20  PRINT
 29  REM - SET MAXIMUM SAMPLE SIZE TO X(M),Y(N)  (WHERE M=MAXIMUM SIZE O
   F SAMPLE 1, N=MAXIMUM SIZE OF SAMPLE 2)
 30  DIM X(25),Y(25)
 40  DIM N(2)
 49  REM - INPUT THE TWO SAMPLES
 50  FOR I=1  TO 2
 60  PRINT "SAMPLE";I;"."
 70  PRINT "  SIZE";
 80  INPUT N(I)
 90  FOR J=1  TO N(I)
100  PRINT "   DATA";J;
110  INPUT Y(J)
120  NEXT J
129  REM - SORT EACH SAMPLE
130  FOR J=1  TO N(I)
140  FOR K=1  TO N(I)-J
150  C=Y(K)
170  IF Y(K)<Y(K+1)  THEN  200
180  Y(K)=Y(K+1)
190  Y(K+1)=C
200  NEXT K
210  NEXT J
220  PRINT
229  REM - TRANSFER FIRST SAMPLE TO X-ARRAY
230  IF I=2  THEN  270
240  FOR J=1  TO N(1)
250  X(J)=Y(J)
260  NEXT J
270  NEXT I
279  REM - ADD UP RANKS
280  R=1
290  I=0
300  J=0
310  I=I+1
320  J=J+1
330  IF I>N(1)  THEN  580
340  IF J>N(2)  THEN  620
350  IF X(I)<Y(J)  THEN  620
360  IF Y(J)<X(I)  THEN  530
369  REM - LINES 370-570 HANDLE EQUAL SCORES FROM BOTH SAMPLES
370  K=2
380  M=I
390  L=J
400  R1=2*R+1
410  R=R+2
420  I=I+1
430  J=J+1
440  IF I>N(1)  THEN  480
450  IF X(I)<>X(I-1)  THEN  480
460  I=I+1
470  GOTO  510
480  IF J>N(2)  THEN  550
```

```
490  IF Y(J)<>Y(J-1)  THEN  550
500  J=J+1
510  R1=R1+R
520  R=R+1
530  K=K+1
540  GOTO  440
550  X=X+(I-M)*R1/K
560  Y=Y+(J-L)*R1/K
570  GOTO  330
580  IF J>N(2)  THEN  660
590  Y=Y+R
600  J=J+1
610  GOTO  640
620  X=X+R
630  I=I+1
640  R=R+1
650  GOTO  330
659  REM - U1=NUMBER OF TIMES SAMPLE 1 SCORES PRECEDE SAMPLE 2 SCORE
660  U1=N(1)*N(2)+N(1)*(N(1)+1)/2-X
669  REM - U2=NUMBER OF TIMES SAMPLE 2 SCORES PRECEDE SAMPLE 1 SCORE
670  U2=N(1)*N(2)+N(2)*(N(2)+1)/2-Y
680  PRINT
690  PRINT "FIRST SAMPLE PRECEDING, U =";U1
700  PRINT "SECOND SAMPLE PRECEDING, U =";U2
710  END
```

Mean, Variance, Standard Deviation

This program calculates the arithmetic mean, variance and standard deviation of grouped or ungrouped data. The data may represent the entire population or just a sample.

Examples:

There are ten people in a hotel lobby, aged 87, 53, 35, 42, 9, 48, 51, 60, 39 and 44. What would the mean, variance and standard deviation of the ages of all the people in the hotel be using the people in the lobby as a sample?

Find the mean, variance and standard deviation of the ages of the cream cheese on a market shelf. The table below lists the age distribution of 50 packages. Assume the table shows the store's entire inventory. What if it is only a sample of the inventory?

age	1	2	3	4	5	6
quantity	15	10	9	6	7	3

cream cheese

```
:RUN
MEAN, VARIANCE, STANDARD DEVIATION

WHICH METHOD (0=POPULATION,1=SAMPLE)? 1
KIND OF DATA (0=GROUPED,1=UNGROUPED)? 1
NUMBER OF OBSERVATIONS? 10
ITEM 1 ? 87
ITEM 2 ? 53
ITEM 3 ? 35
ITEM 4 ? 42
ITEM 5 ? 9
ITEM 6 ? 48
ITEM 7 ? 51
ITEM 8 ? 60
ITEM 9 ? 39
ITEM 10 ? 44

MEAN              VARIANCE          STANDARD DEVIATION
 46.8              389.7333333333   19.741664908

MORE DATA (1=YES, 0=NO)? 1

WHICH METHOD (0=POPULATION,1=SAMPLE)? 0
KIND OF DATA (0=GROUPED,1=UNGROUPED)? 0
NUMBER OF OBSERVATIONS? 6
ITEM, FREQUENCY 1 ? 1,15
ITEM, FREQUENCY 2 ? 2,10
ITEM, FREQUENCY 3 ? 3,9
ITEM, FREQUENCY 4 ? 4,6
ITEM, FREQUENCY 5 ? 5,7
ITEM, FREQUENCY 6 ? 6,3

MEAN              VARIANCE          STANDARD DEVIATION
 2.78              2.5716            1.6036209028
```

```
MORE DATA (1=YES, 0=NO)? 1

WHICH METHOD (0=POPULATION,1=SAMPLE)? 1
KIND OF DATA (0=GROUPED,1=UNGROUPED)? 0
NUMBER OF OBSERVATIONS? 6
ITEM, FREQUENCY 1 ? 1,15
ITEM, FREQUENCY 2 ? 2,10
ITEM, FREQUENCY 3 ? 3,9
ITEM, FREQUENCY 4 ? 4,6
ITEM, FREQUENCY 5 ? 5,7
ITEM, FREQUENCY 6 ? 6,3

MEAN              VARIANCE          STANDARD DEVIATION
 2.78              2.624081632653   1.6199017355

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "MEAN, VARIANCE, STANDARD DEVIATION"
 20  PRINT
 30  PRINT "WHICH METHOD (0=POPULATION,1=SAMPLE)";
 40  INPUT S
 50  PRINT "KIND OF DATA (0=GROUPED,1=UNGROUPED)";
 60  INPUT K
 70  PRINT "NUMBER OF OBSERVATIONS";
 80  INPUT N
 90  R=0
100  M=0
110  P=0
120  IF K=1  THEN  230
129  REM - FOR GROUPED DATA
130  FOR I=1  TO N
140  PRINT "ITEM, FREQUENCY";I;
150  INPUT A,B
159  REM - ACCUMULATE ENTERED VALUES
160  R=R+B*A
169  REM - ACCUMULATE INTERMEDIATE VALUES FOR VARIANCE
170  P=P+B
180  M=M+B*A↑2
190  NEXT I
199  REM - CALCULATE MEAN AND VARIANCE
200  R=R/P
210  V=(M-P*R↑2)/(P-S)
219  REM - PRINT RESULTS
220  GOTO  310
229  REM - FOR UNGROUPED DATA
230  FOR I=1  TO N
240  PRINT "ITEM";I;
250  INPUT D
259  REM - ACCUMULATE ENTERED VALUES
260  P=P+D
269  REM - ACCUMULATE INTERMEDIATE VALUES FOR VARIANCE
270  M=M+D↑2
```

```
NEXT I
REM - CALCULATE MEAN AND VARIANCE, PRINT
R=P/N
V=(M-N*R↑2)/(N-S)
PRINT
REM - PRINT RESULTS
PRINT "MEAN","VARIANCE","STANDARD DEVIATION"
PRINT R,V,SQR(V)
PRINT
REM - RESTART OR END PROGRAM?
PRINT "MORE DATA (1=YES, 0=NO)";
INPUT S
IF S=1  THEN   20
END
```

Geometric Mean and Deviation

This program computes the geometric mean and standard deviation of a set of data.

Example:

Find the geometric mean and standard deviation of 3, 5, 8, 3, 7, 2.

```
:RUN
GEOMETRIC MEAN AND DEVIATION

(TO END PROGRAM ENTER 0 OBSERVATIONS)
NUMBER OF OBSERVATIONS? 6
ITEM 1 ? 3
ITEM 2 ? 5
ITEM 3 ? 8
ITEM 4 ? 3
ITEM 5 ? 7
ITEM 6 ? 2
GEOMETRIC MEAN = 4.140680833732
GEOMETRIC DEVIATION = 1.723689564961

NUMBER OF OBSERVATIONS? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "GEOMETRIC MEAN AND DEVIATION"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER 0 OBSERVATIONS)"
 40  PRINT "NUMBER OF OBSERVATIONS";
 50  INPUT N
 59  REM - TEST FOR END OF PROGRAM
 60  IF N=0  THEN  200
 69  REM - COMPUTE WHICH ROOT TO USE
 70  P=1/N
 80  M=1
 90  FOR I=1  TO N
100  PRINT "ITEM";I;
110  INPUT D
119  REM - ITERATIVELY COMPUTE MEAN
120  M=M*D↑P
129  REM - ACCUMULATE  INTERMEDIATE TERM FOR DEVIATION
130  Q=Q+LOG(D)↑2
140  NEXT I
149  REM - COMPUTE DEVIATION
150  R=EXP(SQR(Q/(N-1)-(N/(N-1)*(LOG(M))↑2)))
160  PRINT "GEOMETRIC MEAN =";M
170  PRINT "GEOMETRIC DEVIATION =";R
180  PRINT
189  REM - RESTART PROGRAM
190  GOTO   40
200  END
```

Binomial Distribution

This program calculates the probability of obtaining a given number of successes in a given number of Bernoulli trials. You must provide the probability of success on a single trial.

Examples:

What is the probability of getting three heads in five tosses of a fair coin?

What is the probability that in five rolls of a fair die, a one (1) appears twice?

```
:RUN
BINOMIAL DISTRIBUTION

(TO END PROGRAM ENTER 0)
NUMBER OF TRIALS? 5
EXACT NUMBER OF SUCCESSES? 3
PROBABILITY OF SUCCESS? .5
PROBABILITY OF 3 SUCCESSES IN 5 TRIALS = .3124999999998

NUMBER OF TRIALS? 5
EXACT NUMBER OF SUCCESSES? 2
PROBABILITY OF SUCCESS? .166666667
PROBABILITY OF 2 SUCCESSES IN 5 TRIALS = .1607510282571

NUMBER OF TRIALS? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "BINOMIAL DISTRIBUTION"
 20  PRINT
 30  DIM M(3)
 40  PRINT "(TO END PROGRAM ENTER 0)"
 50  PRINT "NUMBER OF TRIALS";
 60  INPUT N
 70  IF N=0  THEN  270
 80  PRINT "EXACT NUMBER OF SUCCESSES";
 90  INPUT X
100  PRINT "PROBABILITY OF SUCCESS";
110  INPUT P
119  REM - COMPUTE THE FACTORIALS
120  M(1)=N
130  M(2)=X
140  M(3)=N-X
150  FOR I=1  TO 3
160  IF M(I)=0  THEN  220
170  A=1
180  FOR J=1  TO M(I)
190  A=A*J
200  NEXT J
210  M(I)=LOG(A)
```

```
220  NEXT I
229  REM - USING THE COMPUTED FACTORIALS, COMPUTE PROBABILITY
230  R=EXP(M(1)-M(2)-M(3)+X*LOG(P)+(N-X)*LOG(1-P))
240  PRINT "PROBABILITY OF";X;"SUCCESSES IN";N;"TRIALS =";R
250  PRINT
260  GOTO   50
270  END
```

Poisson Distribution

Using the Poisson distribution this program calculates the probability of an event occurring a given number of times. You must know the expected frequency of the event.

Example:

2000 people are injected with a serum. The probability of any one person having a bad reaction is .001. Thus we can expect two (.001•2000=2) individuals will suffer a bad reaction. What is the probability that four people will have bad reactions? Only one person?

```
;RUN
POISSON DISTRIBUTION

(TO END PROGRAM ENTER 0)
CALCULATED FREQUENCY? 2
TEST FREQUENCY? 4
PROBABILITY OF 4 OCCURRENCES = 9.02235221E-02

CALCULATED FREQUENCY? 2
TEST FREQUENCY? 1
PROBABILITY OF 1 OCCURRENCES = .270670566473

CALCULATED FREQUENCY? 0

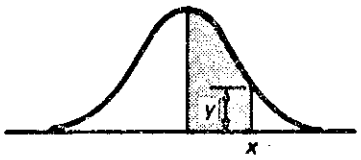
END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "POISSON DISTRIBUTION"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER 0)"
 40  PRINT "CALCULATED FREQUENCY";
 50  INPUT L
 59  REM - END PROGRAM?
 60  IF L=0  THEN  180
 70  PRINT "TEST FREQUENCY";
 80  INPUT X
 89  REM - COMPUTE FACTORIAL
 90  A=1
100  FOR I=1  TO X
110  A=A*I
120  NEXT I
129  REM - COMPUTE PROBABILITY
130  A=LOG(A)
140  A=EXP(-L+X*LOG(L)-A)
150  PRINT "PROBABILITY OF";X;"OCCURRENCES =";A
160  PRINT
169  REM - RESTART PROGRAM
170  GOTO   40
180  END
```

Normal Distribution

This program calculates the probability and frequency of given values on a standard normal distribution curve. You can use non-standard variables if you know the mean and standard deviation.



Standard normal distribution

The shaded area represents the probability of x. y corresponds to the frequency of x.

The normal probability is approximated using the following formula:

$$\text{probability} = 1 - r(a_1 t + a_2 t^2 + a_3 t^3) + \epsilon(x)$$

where:
$$a_1 = .4361836$$
$$a_2 = -.1201676$$
$$a_3 = .9372980$$
$$r = (e^{-x^2/2})(2\pi)^{-1/2}$$
$$t = (1 + .3326x)^{-1}$$
$$|\epsilon(x)| < 10^{-5}$$

Example:

The mean weight of the male students at a college is 150 pounds. The standard deviation is 15 pounds. If the weights are normally distributed, what is the probability that a student weighs between 150 and 180 pounds? Between 130 and 150 pounds?

```
:RUN
NORMAL DISTRIBUTION

(0=STANDARD, 1=NON-STANDARD)
WHICH TYPE OF VARIABLE? 1
MEAN? 150
STANDARD DEVIATION? 15

TO END PROGRAM ENTER X=99999
X =? 180
FREQUENCY = 5.39909665E-02
PROBABILITY = .977241189885

X =? 130
FREQUENCY = .1640100746762
PROBABILITY = .908798074993

X =? 99999

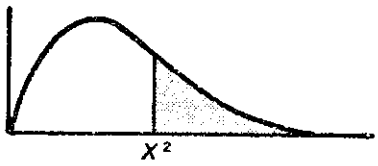
END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "NORMAL DISTRIBUTION"
 20  PRINT
 30  PRINT "(0=STANDARD, 1=NON-STANDARD)"
 40  PRINT "WHICH TYPE OF VARIABLE";
 50  INPUT S
 60  IF S=0  THEN  120
 69  REM - LINES 70-110 REQUEST 'NON-STANDARD' VARIABLE DATA
 70  PRINT "MEAN";
 80  INPUT M
 90  PRINT "STANDARD DEVIATION";
100  INPUT S
110  GOTO  130
120  S=1
130  PRINT
140  PRINT "TO END PROGRAM ENTER X=99999"
150  PRINT "X =";
160  INPUT Y
170  IF Y=99999  THEN  290
179  REM - ADJUST FOR NON-STANDARD VARIABLES
180  Y=ABS((Y-M)/S)
189  REM - COMPUTE FREQUENCY (Y COORDINATE)
190  R=EXP(-(Y↑2)/2)/2.5066282746
200  PRINT "FREQUENCY =";R
210  Z=Y
219  REM - APPROXIMATE PROBABILITY (AREA UNDER CURVE)
220  Y=1/(1+.33267*ABS(Y))
230  T=1-R*(.4361836*Y-.1201676*Y↑2+.937298*Y↑3)
239  REM - ADJUST FOR NEGATIVE VARIABLES
240  IF Z>=0  THEN  260
250  T=1-T
260  PRINT "PROBABILITY =";T
270  PRINT
280  GOTO  150
290  END
```

Chi-square Distribution

This program calculates the tail-end value for points on a chi-square (X^2) distribution curve. You must provide the value of X^2 and the degrees of freedom.



Chi-square distribution

The shaded area represents the tail-end value of X^2.

The X^2 distribution function is calculated using the following formulas:

$$\text{with } v \text{ odd, tail-end value} = 1 - \frac{(X^2)(v+1)/2 \cdot e^{-x^2/2}}{1 \cdot 3 \cdot 5 \ldots \cdot v} \cdot \left(\frac{2}{X^2 \pi}\right)^{1/2} \cdot Z$$

$$\text{with } v \text{ even, tail-end value} = 1 - \frac{(X^2)v/2 \cdot e^{-x^2/2}}{2 \cdot 4 \cdot \ldots \cdot v} \cdot Z$$

where: v = degrees of freedom

$$Z = 1 + \sum_{m=1}^{\infty} \frac{(X^2)m}{(v+2) \cdot (v+4) \cdot \ldots (v+2m)}$$

Since the summation in the calculation of Z cannot actually extend to infinity, we stop summation when the next term is less than a chosen level of precision. The computational precision is limited to approximately 10^{-7}.

Example:

Of a group of 168 people who complained they did not sleep well, 54 were given sleeping pills and the remainder received placebos. They were later asked whether or not the pills had helped them sleep. The X^2 statistic for this study was computed to be 2.571108 with one degree of freedom. What is the tail-end value?

```
:RUN
CHI-SQUARE DISTRIBUTION

(TO END PROGRAM ENTER 0)
DEGREES OF FREEDOM? 1
CHI-SQUARE? 2.571108
TAIL END VALUE = .108831484618

DEGREES OF FREEDOM? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "CHI-SQUARE DISTRIBUTION"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER 0)"
 40  PRINT "DEGREES OF FREEDOM";
 50  INPUT V
 60  IF V=0  THEN  280
 70  PRINT "CHI-SQUARE";
 80  INPUT W
 89  REM - R=THE DENOMINATOR PRODUCT
 90  R=1
100  FOR I=V  TO 2  STEP -2
110  R=R*I
120  NEXT I
129  REM - K=THE NUMERATOR PRODUCT
130  K=W↑(INT((V+1)/2))*EXP(-W/2)/R
139  REM - THE PI FACTOR IS USED ONLY WHEN DEG. FREEDOM ARE ODD
140  IF INT(V/2)=V/2  THEN  170
150  J=SQR(2/W/3.141592653599)
160  GOTO  180
169  REM - L (SUMMATION FACTOR) CALCULATED LINES 170-240
170  J=1
180  L=1
190  M=1
200  V=V+2
210  M=M*W/V
219  REM - CHECK FOR END OF SUMMATION
220  IF M<.0000001  THEN  250
230  L=L+M
240  GOTO  200
250  PRINT "TAIL END VALUE =";1-J*K*L
260  PRINT
270  GOTO  40
280  END
```

OPTION

You may wish to compute the percentile rather than the tail-end value. This value corresponds to the unshaded area in the figure above. The program changes necessary are listed following the example below.

Example:

What is the percentile in the example above?

```
>RUN
CHI-SQUARE DISTRIBUTION

(TO END PROGRAM ENTER 0)
DEGREES OF FREEDOM? 1
CHI-SQUARE? 2.571108
PERCENTILE =  .8911685153823

DEGREES OF FREEDOM? 0

END PROGRAM
```

PROGRAM LISTING

```
   1  REM - OPTION 250
  10  PRINT "CHI-SQUARE DISTRIBUTION"
   :
 240  GOTO  200
 250  PRINT "PERCENTILE =";J*K*L
 260  PRINT
 270  GOTO   40
 280  END
```

Chi-square Test

This program calculates the chi-square (X^2) statistic and degrees of freedom associated with a given contingency table. The expected value for each cell and X^2 contribution from each cell are also printed.

The dimension statement at line 30 limits the size of the contingency table. You can change the dimensions according to the following scheme:

```
30 DIM V1(R*C), V2(C), A(R)
```

where: R = number of rows in the contingency table
C = number of columns in the contingency table

Example:

Of a group of people who complained they could not sleep well, some were given sleeping pills while others were given placebos. Later they were asked whether or not the pills had helped them sleep. The results are detailed in the table below. What is the value of the X^2 statistic?

	slept well	slept poorly
sleeping pill	44	10
placebo	81	35

```
:30 DIM V1(4),V2(2),A(2)
:RUN
CHI-SQUARE TEST

NUMBER OF ROWS? 2
NUMBER OF COLUMNS? 2
CONTINGENCY TABLE:
ROW 1
  ELEMENT 1 ? 44
  ELEMENT 2 ? 10
ROW 2
  ELEMENT 1 ? 81
  ELEMENT 2 ? 35

OBSERVED VALUE  EXPECTED VALUE  CHI^2 CONTRIBUTION
  COLUMN 1
     44          39.70588235294   .3625490196148
     81          85.29411764706   .1687728194759
  COLUMN 2
     10          14.29411764706   1.007080610041
     35          30.70588235294   .4688133887433

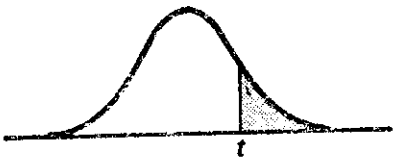
CHI-SQUARE = 2.007215836564
DEGREES OF FREEDOM = 1
```

PROGRAM LISTING

```
 10  PRINT "CHI-SQUARE TEST"
 20  PRINT
 28  REM - LIMIT SIZE OF CONTINGENCY TABLES TO V1(R*C),V2(C),A(R)
 29  REM - WHERE R=NO. OF ROWS, C=NO. OF COLUMNS
 30  DIM V1(25),V2(5),A(5)
 40  PRINT "NUMBER OF ROWS";
 49  REM - LINES 50-150 INPUT CONTINGENCY TABLE
 50  INPUT R
 60  PRINT "NUMBER OF COLUMNS";
 70  INPUT C
 80  PRINT "CONTINGENCY TABLE:"
 90  FOR I=1  TO R
100  PRINT "ROW";I
110  FOR J=1  TO C
120  PRINT "  ELEMENT";J;
130  INPUT V1((I-1)*C+J)
140  NEXT J
150  NEXT I
160  PRINT
169  REM - ADD UP MARGINAL FREQUENCIES FOR EACH ROW
170  L=0
180  M=1
190  FOR I=1  TO R
200  FOR J=1  TO C
210  A(I)=A(I)+V1(M)
220  M=M+1
230  NEXT J
240  L=L+A(I)
250  NEXT I
260  N=R*C
269  REM - ADD UP MARGINAL FREQUENCIES FOR EACH COLUMN
270  FOR I=1  TO C
280  FOR J=I  TO N  STEP C
290  V2(I)=V2(I)+V1(J)
300  NEXT J
310  NEXT I
320  Z=0
330  PRINT "OBSERVED VALUE","EXPECTED VALUE","CHI↑2 CONTRIBUTION"
340  FOR I=1  TO C
350  PRINT "  COLUMN";I
360  FOR J=1  TO R
369  REM - P=EXPECTED CELL VALUE
370  P=A(J)*V2(I)/L
375  X=I+(J-1)*C
379  REM  - USE YATES' CORRECTION FOR CONTINUITY IN 2 X 2 CHI-SQUARE
     TESTS
380  IF R<>2  THEN  390
381  IF  C<>2  THEN  390
382  Y=(ABS(V1(X)-P)-.5)↑2/P
383  GOTO  400
389  REM - Y=CHI-SQUARE CONTRIBUTION FROM THIS CELL
390  Y=(V1(X)-P)↑2/P
399  REM - Z=TOTAL CHI-SQUARE VALUE
400  Z=Z+Y
410  PRINT "     ";V1(X),P,Y
420  NEXT J
430  NEXT I
440  PRINT
450  PRINT "CHI-SQUARE =";Z
460  PRINT "DEGREES OF FREEDOM =";(C-1)*(R-1)
470  END
```

Student's t -distribution

This program calculates right-tail values for points on a t -distribution curve. You must provide the value of t and the degrees of freedom.



Student's t -distribution

The shaded area represents the right-tail value for t.

The right-tail value is approximated using the following formula:

$$\text{right-tail value} = \frac{1}{4}(1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4)^{-4} + \epsilon(x)$$

where: $a_1 = .196854$
$a_2 = .115194$
$a_3 = .000344$
$a_4 = .019527$
$x = \left(t^{2/3}\left(1 - \frac{2}{9d}\right) - \frac{7}{9}\right)\left(\frac{2}{9} + t^{4/3} \cdot \frac{2}{9d}\right)^{-1/2}$
$d =$ degrees of freedom

$|\epsilon(x)| < 2.5 \cdot 10^{-4}$

Examples:

What is the right-tail value when the t -value is 2.921 and there are 16 degrees of freedom?

What is the right-tail value when the t -value is 11.178 and there are 5 degrees of freedom?

```
:RUN
STUDENT'S T-DISTRIBUTION

(TO END PROGRAM ENTER A T-VALUE OF 0)
T-VALUE? 2.921
DEGREES OF FREEDOM? 16
RIGHT TAIL VALUE = 4.90000000E-03

T-VALUE? 11.178
DEGREES OF FREEDOM? 5
RIGHT TAIL VALUE = 2.00000000E-04

T-VALUE? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "STUDENT'S T-DISTRIBUTION"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER A T-VALUE OF 0)"
 40  PRINT "T-VALUE";
 50  INPUT T
 60  IF T=0  THEN  340
 70  PRINT "DEGREES OF FREEDOM";
 80  INPUT D
 90  X=1
100  Y=1
110  T=T↑2
119  REM - COMPUTE USING INVERSE FOR SMALL T-VALUES
120  IF T<1  THEN  170
130  S=Y
140  R=D
150  Z=T
160  GOTO  200
170  S=D
180  R=Y
190  Z=1/T
200  J=2/9/S
210  K=2/9/R
219  REM - COMPUTE USING APPROXIMATION FORMULAS
220  L=ABS((1-K)*Z↑(1/3)-1+J)/SQR(K*Z↑(2/3)+J)
230  IF R<4  THEN  270
240  X=.25/(1+L*(.196854+L*(.115194+L*(.000344+L*.019527))))↑4
250  X=INT(X*10000+.5)/10000
260  GOTO  290
270  L=L*(1+.08*L↑4/R↑3)
280  GOTO  240
289  REM - ADJUST IF INVERSE WAS COMPUTED
290  IF T>=1  THEN  310
300  X=1-X
310  PRINT "RIGHT TAIL VALUE =";X
320  PRINT
330  GOTO   40
340  END
```

Student's *t*-distribution Test

This program calculates the *t*-statistic and degrees of freedom for Student's distribution. The calculations can be based on any one of three hypotheses.

The first hypothesis assumes that one population mean is equal to a given value. You must enter the elements of the sample and the value of the mean.

The remaining hypotheses compare two populations. In both tests the means of the two populations are equal, but the standard deviations may be equal or unequal. For these hypotheses you must enter the elements of each sample.

The dimension statement at line 30 limits the size of the samples you may enter. You can change the limit according to the following scheme:

```
30 DIM P(N,2)
```

where N = maximum sample size.

Examples:

A sample of children's IQ's was taken, the results being 101, 99, 120, 79, 111, 98, 106, 112, 87, and 97. Calculate the *t*-statistic assuming the population mean is 100.

A second sample was taken, the results being 101, 95, 130, 150, 75, 79, 111, 100, 98 and 91. Calculate the *t*-statistic based on the hypothesis that the two samples have equal means and standard deviations.

```
:RUN
STUDENT'S T-DISTRIBUTION TEST

TEST 1: MEAN=X
TEST 2: MEAN=MEAN,STANDARD DEVIATION=STANDARD DEVIATION
TEST 3: MEAN=MEAN,STANDARD DEVIATION<>STANDARD DEVIATION
WHICH HYPOTHESIS? 1

SAMPLE 1 :
  NUMBER OF ELEMENTS? 10
   ELEMENT 1 ? 101
   ELEMENT 2 ? 99
   ELEMENT 3 ? 120
   ELEMENT 4 ? 79
   ELEMENT 5 ? 111
   ELEMENT 6 ? 98
   ELEMENT 7 ? 106
   ELEMENT 8 ? 112
   ELEMENT 9 ? 87
   ELEMENT 10 ? 97

VALUE OF MEAN? 100

T-VALUE = .261513016411
DEGREES OF FREEDOM = 9

END PROGRAM
```

```
:RUN
STUDENT'S T-DISTRIBUTION TEST

TEST 1: MEAN=X
TEST 2: MEAN=MEAN,STANDARD DEVIATION=STANDARD DEVIATION
TEST 3: MEAN=MEAN,STANDARD DEVIATION<>STANDARD DEVIATION
WHICH HYPOTHESIS? 2

SAMPLE 1 :
  NUMBER OF ELEMENTS? 10
   ELEMENT 1 ? 101
   ELEMENT 2 ? 99
   ELEMENT 3 ? 120
   ELEMENT 4 ? 79
   ELEMENT 5 ? 111
   ELEMENT 6 ? 98
   ELEMENT 7 ? 106
   ELEMENT 8 ? 112
   ELEMENT 9 ? 87
   ELEMENT 10 ? 97
SAMPLE 2 :
  NUMBER OF ELEMENTS? 10
   ELEMENT 1 ? 101
   ELEMENT 2 ? 95
   ELEMENT 3 ? 130
   ELEMENT 4 ? 150
   ELEMENT 5 ? 75
   ELEMENT 6 ? 79
   ELEMENT 7 ? 111
   ELEMENT 8 ? 100
   ELEMENT 9 ? 98
   ELEMENT 10 ? 91


T-VALUE = .246515212849
DEGREES OF FREEDOM = 18

END PROGRAM

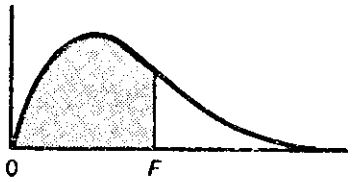

PROGRAM LISTING

  10  PRINT "STUDENT'S T-DISTRIBUTION TEST"
  20  PRINT
  29  REM - LIMIT SAMPLE SIZE TO P(N,2)  WHERE N=MAX. SAMPLE SIZE
  30  DIM P(10,2)
  40  DIM V(2),R(2),M(2),D(2)
  50  PRINT "TEST 1: MEAN=X"
  60  PRINT "TEST 2: MEAN=MEAN,STANDARD DEVIATION=STANDARD DEVIATION"
  70  PRINT "TEST 3: MEAN=MEAN,STANDARD DEVIATION<>STANDARD DEVIATION"
  80  PRINT "WHICH HYPOTHESIS";
  90  INPUT T
 100  PRINT
 109  REM - INPUT 1 OR 2 SAMPLES DEPENDING ON HYPOTHESIS
 110  FOR I=1  TO SGN(T-1)+1
 120  V(I)=0
 130  D(I)=0
```

```
140  PRINT "SAMPLE";I;":"
150  PRINT "  NUMBER OF ELEMENTS";
160  INPUT R(I)
170  FOR J=1  TO R(I)
180  PRINT "   ELEMENT";J;
190  INPUT P(J,I)
199  REM - ACCUMULATE SAMPLES
200  V(I)=V(I)+P(J,I)
210  D(I)=D(I)+P(J,I)↑2
220  NEXT J
229  REM - COMPUTE INTERMEDIATE VALUES
230  M(I)=V(I)/R(I)
240  V(I)=(D(I)-V(I)↑2/R(I))/(R(I)-1)
250  NEXT I
260  PRINT
270  IF T=2  THEN  340
280  IF T=3  THEN  380
289  REM - INPUT GIVEN VALUE FOR FIRST HYPOTHESIS
290  PRINT "VALUE OF MEAN";
300  INPUT M
309  REM - COMPUTE T AND DEGREES OF FREEDOM FOR FIRST HYPOTHESIS
310  A=(M(1)-M)*SQR(R(1)/V(1))
320  B=R(1)-1
330  GOTO  420
339  REM - COMPUTE T AND DEGREES OF FREEDOM FOR SECOND HYPOTHESIS
340  A=(M(1)-M(2))/SQR(1/R(1)+1/R(2))
350  B=R(1)+R(2)-2
360  A=A/SQR(((R(1)-1)*V(1)+(R(2)-1)*V(2))/B)
370  GOTO  420
379  REM - COMPUTE T AND DEGREES OF FREEDOM FOR THIRD HYPOTHESIS
380  A=(M(1)-M(2))/SQR(V(1)/R(1)+V(2)/R(2))
390  B=(V(1)/R(1)+V(2)/R(2))↑2
400  B=B/((V(1)/R(1))↑2/(R(1)+1)+(V(2)/R(2))↑2/(R(2)+1))-2
410  B=INT(B+.5)
420  PRINT
430  PRINT "T-VALUE =";ABS(A)
440  PRINT "DEGREES OF FREEDOM =";B
450  END
```

F -distribution

This program calculates percentile values for given values on an *F* -distribution curve. You must provide the value of *F*, the degrees of freedom in the numerator and the degrees of freedom in the denominator.



The *F* -distribution

The area of the shaded region represents the percentile.

The *F* -distribution function is approximated using the following formula:

$$\text{percentile} = 1 - \frac{1}{2}(1 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4)^{-4} + \epsilon(y)$$

where: $a_1 = .196854$
$a_2 = .115194$
$a_3 = .000344$
$a_4 = .019527$

$$y = \left(F^{1/3}\left(1 - \frac{2}{9d_2}\right) - \left(1 - \frac{2}{9d_1}\right)\right)\left(\frac{2}{9d_1} + F^{2/3} \cdot \frac{2}{9d_2}\right)^{-1/2}$$

d_1 = degrees of freedom in numerator
d_2 = degrees of freedom in denominator
$|\epsilon(y)| < 2.5 \times 10^{-4}$

Examples:

What is the percentile on an *F* -distribution curve when the *F* -value is .474 and the degrees of freedom are 1 and 18?

What is the percentile when the *F* -value is 23.7 and the degrees of freedom are 3 and 6?

```
:RUN
F-DISTRIBUTION

(TO END PROGRAM ENTER AN F-VALUE OF 0)
F-VALUE? .474
DEGREES OF FREEDOM IN NUMERATOR? 1
DEGREES OF FREEDOM IN DENOMINATOR? 18
PERCENTILE = .4937

F-VALUE? 23.7
DEGREES OF FREEDOM IN NUMERATOR? 3
DEGREES OF FREEDOM IN DENOMINATOR? 6
PERCENTILE = .9984

F-VALUE? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "F-DISTRIBUTION"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER AN F-VALUE OF 0)"
 40  PRINT "F-VALUE";
 50  INPUT F
 60  IF F=0  THEN  340
 70  PRINT "DEGREES OF FREEDOM IN NUMERATOR";
 80  INPUT D1
 90  PRINT "DEGREES OF FREEDOM IN DENOMINATOR";
100  INPUT D2
110  X=1
119  REM - COMPUTE USING INVERSE FOR SMALL F-VALUES
120  IF F<1  THEN  170
130  S=D1
140  T=D2
150  Z=F
160  GOTO  200
170  S=D2
180  T=D1
190  Z=1/F
200  J=2/9/S
210  K=2/9/T
219  REM - COMPUTE USING APPROXIMATION FORMULAS
220  Y=ABS((1-K)*Z↑(1/3)-1+J)/SQR(K*Z↑(2/3)+J)
230  IF T<4  THEN  270
240  X=.5/(1+Y*(.196854+Y*(.115194+Y*(.000344+Y*.019527))))↑4
250  X=INT(X*10000+.5)/10000
260  GOTO  290
270  Y=Y*(1+.08*Y↑4/T↑3)
280  GOTO  240
289  REM - ADJUST IF INVERSE WAS COMPUTED
290  IF F>=1  THEN  310
300  X=1-X
310  PRINT "PERCENTILE =";1-X
320  PRINT
330  GOTO  40
340  END
```

OPTION

You may prefer to compute the tail-end value (the area of the unshaded region in the figure above). The program changes necessary are listed following the examples below.

Examples:

What is the tail-end value on an *F* -distribution curve when the *F* -value is .474 and the degrees of freedom are 1 and 18?

What is the tail-end value when the *F* -value is 23.7 and the degrees of freedom are 3 and 6?

```
:RUN
F-DISTRIBUTION

(TO END PROGRAM ENTER AN F-VALUE OF 0)
F-VALUE? .474
DEGREES OF FREEDOM IN NUMERATOR? 1
DEGREES OF FREEDOM IN DENOMINATOR? 18
TAIL END VALUE = .5063

F-VALUE? 23.7
DEGREES OF FREEDOM IN NUMERATOR? 3
DEGREES OF FREEDOM IN DENOMINATOR? 6
TAIL END VALUE = 1.60000000E-03

F-VALUE? 0

END PROGRAM
```

PROGRAM LISTING

```
   1  REM - OPTION 310
  10  PRINT "F-DISTRIBUTION"
   :
 300  X=1-X
 310  PRINT "TAIL END VALUE =";X
 320  PRINT
 330  GOTO   40
 340  END
```

Linear Correlation Coefficient

This program computes the coefficient of correlation between two variables. A linear relationship is assumed between the variables. You must enter the coordinates of a group of data points forming the regression line.

Example:

The height of twelve men and their sons is recorded in the table below. What is the coefficient of correlation between the heights of fathers and the heights of their sons?

father	65	63	67	64	68	62	70	66	68	67	69	71
son	68	66	68	65	69	66	68	65	71	67	68	70

height in inches

```
:RUN
LINEAR CORRELATION COEFFICIENT

NUMBER OF POINTS? 12
X,Y OF POINT 1 ? 65,68
X,Y OF POINT 2 ? 63,66
X,Y OF POINT 3 ? 67,68
X,Y OF POINT 4 ? 64,65
X,Y OF POINT 5 ? 68,69
X,Y OF POINT 6 ? 62,66
X,Y OF POINT 7 ? 70,68
X,Y OF POINT 8 ? 66,65
X,Y OF POINT 9 ? 68,71
X,Y OF POINT 10 ? 67,67
X,Y OF POINT 11 ? 69,68
X,Y OF POINT 12 ? 71,70

COEFFICIENT OF CORRELATION = .7026516450773

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "LINEAR CORRELATION COEFFICIENT"
 20  PRINT
 30  PRINT "NUMBER OF POINTS";
 40  INPUT N
 50  J=0
 60  K=0
 70  L=0
 80  M=0
 90  R=0
 99  REM - ENTER COORDINATES OF DATA POINTS
100  FOR I=1  TO N
110  PRINT "X,Y OF POINT";I;
120  INPUT X,Y
129  REM - ACCUMULATE INTERMEDIATE VALUES
```

```
130  J=J+X
140  K=K+Y
150  L=L+X↑2
160  M=M+Y↑2
170  R=R+X*Y
180  NEXT I
189  REM - CALCULATE COEFFICIENT, PRINT
190  R2=(N*R-J*K)/SQR((N*L-J↑2)*(N*M-K↑2))
200  PRINT
210  PRINT "COEFFICIENT OF CORRELATION =";R2
220  END
```

Linear Regression

This program fits a straight line to a given set of coordinates using the method of least squares. The equation of the line, coefficient of determination, coefficient of correlation and standard error of estimate are printed. Once the line has been fitted, you may predict values of *y* for given values of *x*.

Example:

The table below shows the height and weight of 11 male college students. Fit a curve to these points. How much would the average 70" and 72" male student weigh?

height (in.)	71	73	64	65	61	70	65	72	63	67	64
weight (lbs.)	160	183	154	168	159	180	145	210	132	168	141

```
:RUN
LINEAR REGRESSION

NUMBER OF KNOWN POINTS? 11
X,Y OF POINT 1 ? 71,160
X,Y OF POINT 2 ? 73,183
X,Y OF POINT 3 ? 64,154
X,Y OF POINT 4 ? 65,168
X,Y OF POINT 5 ? 61,159
X,Y OF POINT 6 ? 70,180
X,Y OF POINT 7 ? 65,145
X,Y OF POINT 8 ? 72,210
X,Y OF POINT 9 ? 63,132
X,Y OF POINT 10 ? 67,168
X,Y OF POINT 11 ? 64,141

F(X) =-106.7916666666 + ( 4.047222222222 * X )

COEFFICIENT OF DETERMINATION (R^2) = .5562601669757
COEFFICIENT OF CORRELATION = .74582851043
STANDARD ERROR OF ESTIMATE = 15.41348816

INTERPOLATION: (ENTER X=0 TO END PROGRAM)
X =? 70
Y = 176.5138888889

X =? 72
Y = 184.6083333334

X =? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "LINEAR REGRESSION"
 20  PRINT
 30  PRINT "NUMBER OF KNOWN POINTS";
 40  INPUT N
 50  J=0
 60  K=0
 70  L=0
 80  M=0
 90  R2=0
 99  REM - LOOP TO ENTER COORDINATES OF POINTS
100  FOR I=1  TO N
110  PRINT "X,Y OF POINT";I;
120  INPUT X,Y
129  REM - ACCUMULATE INTERMEDIATE SUMS
130  J=J+X
140  K=K+Y
150  L=L+X↑2
160  M=M+Y↑2
170  R2=R2+X*Y
180  NEXT I
189  REM - COMPUTE CURVE COEFFICIENT
190  B=(N*R2-K*J)/(N*L-J↑2)
200  A=(K-B*J)/N
210  PRINT
220  PRINT "F(X) =";A;"+ (";B;"* X )"
229  REM - COMPUTE REGRESSION ANALYSIS
230  J=B*(R2-J*K/N)
240  M=M-K↑2/N
250  K=M-J
260  PRINT
270  R2=J/M
280  PRINT "COEFFICIENT OF DETERMINATION (R↑2) =";R2
290  PRINT "COEFFICIENT OF CORRELATION =";SQR(R2)
300  PRINT "STANDARD ERROR OF ESTIMATE =";SQR(K/(N-2))
310  PRINT
319  REM - ESTIMATE Y-COORDINATES OF POINTS WITH ENTERED X-COORDINATES
320  PRINT "INTERPOLATION: (ENTER X=0 TO END PROGRAM)"
330  PRINT "X =";
340  INPUT X
349  REM - RESTART OR END PROGRAM?
350  IF X=0  THEN  390
360  PRINT "Y =";A+B*X
370  PRINT
380  GOTO  330
390  END
```

Multiple Linear Regression

This program finds the coefficients of a multiple variable linear equation using the method of least squares. The equation is of the following form:

$$y = c + a_1 x_1 + a_2 x_2 + \ldots a_n x_n$$

where: y = dependent variable
c = constant
$a_1, a_2 \ldots a_n$ = coefficients of independent variables $x_1, x_2, \ldots x_n$

The constant and the coefficients are printed.

You must provide the x - and y -coordinates of known data points. Once the equation has been found using the data you enter, you may predict values of the dependent variables for given values of the independent variables.

The dimension statement at line 30 limits the number of known data points the equation may contain. You can change this limit according to the following scheme:

30 DIM X(N+1), S(N+1), T(N+1), A(N+1,N+2)

where N = the number of known data points.

Example:

The table below shows the age, height and weight of eight boys. Using weight as the dependent variable, fit a curve to the data. Estimate the weight of a seven year old boy who is 51 inches tall.

age	8	9	6	10	8	9	9	7
height	48	49	44	59	55	51	55	50
weight	59	55	50	80	61	75	67	58

```
]RUN
MULTIPLE LINEAR REGRESSION

NUMBER OF KNOWN POINTS? 8
NUMBER OF INDEPENDENT VARIABLES? 2
POINT 1
  VARIABLE 1 ? 8
  VARIABLE 2 ? 48
  DEPENDENT VARIABLE? 59
POINT 2
  VARIABLE 1 ? 9
  VARIABLE 2 ? 49
  DEPENDENT VARIABLE? 55
POINT 3
  VARIABLE 1 ? 6
  VARIABLE 2 ? 44
  DEPENDENT VARIABLE? 50
```

```
POINT 4
  VARIABLE 1 ? 10
  VARIABLE 2 ? 59
  DEPENDENT VARIABLE? 80
POINT 5
  VARIABLE 1 ? 8
  VARIABLE 2 ? 55
  DEPENDENT VARIABLE? 61
POINT 6
  VARIABLE 1 ? 9
  VARIABLE 2 ? 51
  DEPENDENT VARIABLE? 75
POINT 7
  VARIABLE 1 ? 9
  VARIABLE 2 ? 55
  DEPENDENT VARIABLE? 67
POINT 8
  VARIABLE 1 ? 7
  VARIABLE 2 ? 50
  DEPENDENT VARIABLE? 58

EQUATION COEFFICIENTS:
     CONSTANT:-15.70212765959
VARIABLE( 1 ): 3.680851063828
VARIABLE( 2 ): .9432624113481

COEFFICIENT OF DETERMINATION (R^2) = .7156973568726
COEFFICIENT OF MULTIPLE CORRELATION = .84598898271
STANDARD ERROR OF ESTIMATE = 6.4288798755

INTERPOLATION: (ENTER 0 TO END PROGRAM)
VARIABLE 1 ? 7
VARIABLE 2 ? 51
DEPENDENT VARIABLE = 58.17021276596

VARIABLE 1 ? 0

END PROGRAM


PROGRAM LISTING

  10  PRINT "MULTIPLE LINEAR REGRESSION"
  20  PRINT
  29  REM - SET ARRAY LIMITS TO X(N+1),S(N+1),T(N+1),A(N+1,N+2)
  30  DIM X(9),S(9),T(9),A(3,10)
  40  PRINT "NUMBER OF KNOWN POINTS";
  50  INPUT N
  60  PRINT "NUMBER OF INDEPENDENT VARIABLES";
  70  INPUT V
  80  X(1)=1
  90  FOR I=1  TO N
 100  PRINT "POINT";I
 110  FOR J=1  TO V
 119  REM - ENTER INDEPENDENT VARIABLES FOR EACH POINT
 120  PRINT "  VARIABLE";J;
 130  INPUT X(J+1)
```

```
140  NEXT J
149  REM - ENTER DEPENDENT VARIABLE FOR EACH POINT
150  PRINT "  DEPENDENT VARIABLE";
160  INPUT X(V+2)
169  REM - POPULATE A MATRIX TO BE USED IN CURVE FITTING
170  FOR K=1  TO V+1
180  FOR L=1  TO V+2
190  A(K,L)=A(K,L)+X(K)*X(L)
200  S(K)=A(K,V+2)
210  NEXT L
220  NEXT K
230  S(V+2)=S(V+2)+X(V+2)↑2
240  NEXT I
248  REM - STATEMENTS 250 TO 500 FIT CURVE BY SOLVING THE SYSTEM OF
249  REM - LINEAR EQUATIONS IN MATRIX A()
250  FOR I=2  TO V+1
260  T(I)=A(1,I)
270  NEXT I
280  FOR I=1  TO V+1
290  J=I
300  IF A(J,I)<>0  THEN  340
305  J=J+1
310  IF J<=V+1  THEN  300
320  PRINT "NO UNIQUE SOLUTION"
330  GOTO  810
340  FOR K=1  TO V+2
350  B=A(I,K)
360  A(I,K)=A(J,K)
370  A(J,K)=B
380  NEXT K
390  Z=1/A(I,I)
400  FOR K=1  TO V+2
410  A(I,K)=Z*A(I,K)
420  NEXT K
430  FOR J=1  TO V+1
440  IF J=I  THEN  490
450  Z=-A(J,I)
460  FOR K=1  TO V+2
470  A(J,K)=A(J,K)+Z*A(I,K)
480  NEXT K
490  NEXT J
500  NEXT I
510  PRINT
520  PRINT "EQUATION COEFFICIENTS:"
525  PRINT "      CONSTANT:";A(1,V+2)
530  FOR I=2  TO V+1
540  PRINT "VARIABLE(";I-1;"):";A(I,V+2)
550  NEXT I
560  P=0
570  FOR I=2  TO V+1
580  P=P+A(I,V+2)*(S(I)-T(I)*S(1)/N)
590  NEXT I
600  R=S(V+2)-S(1)↑2/N
610  Z=R-P
620  L=N-V-1
640  PRINT
650  I=P/R
```

```
660  PRINT "COEFFICIENT OF DETERMINATION (R↑2) =";I
670  PRINT "COEFFICIENT OF MULTIPLE CORRELATION =";SQR(I)
680  PRINT "STANDARD ERROR OF ESTIMATE ="; SQR(ABS(Z/L))
690  PRINT
699  REM - ESTIMATE DEPENDENT VARIABLE FROM ENTERED INDEPENDENT VARIAB
   LES
700  PRINT "INTERPOLATION: (ENTER 0 TO END PROGRAM)"
710  P=A(1,V+2)
720  FOR J=1  TO V
730  PRINT "VARIABLE";J;
740  INPUT X
749  REM - TEST FOR END OF PROGRAM
750  IF X=0  THEN  810
760  P=P+A(J+1,V+2)*X
770  NEXT J
780  PRINT "DEPENDENT VARIABLE =";P
790  PRINT
800  GOTO  710
810  END
```

N th Order Regression

This program finds the coefficients of an *N* th order equation using the method of least squares. The equation is of the following form:

$$y = c + a_1 x + a_2 x^2 + \ldots a_n x^n$$

where: y = dependent variable
c = constant
$a_1, a_2 \ldots a_n$ = coefficients of independent variables $x, x^2, \ldots x^n$, respectively

The equation coefficients, coefficient of determination, coefficient of correlation and standard error of estimate are printed.

You must provide the *x* - and *y* -coordinates for known data points. Once the equation has been computed you may predict values of *y* for given values of *x*.

The dimension statement at line 30 limits the degree of the equation. You can change this limit according to the following scheme:

```
30 DIM A(2*D+1), R(D+1,D+2), T(D+2)
```

where *D* = maximum degree of equation.

Example:

The table below gives the stopping distance (reaction plus braking distance) of an automobile at various speeds. Fit an exponential curve to the data. Estimate the stopping distance at 55 m.p.h.

m.p.h.	20	30	40	50	60	70
stopping distance	54	90	138	206	292	396

```
:30 DIM A(5),R(3,4),T(4)
:RUN
NTH-ORDER REGRESSION

DEGREE OF EQUATION? 2
NUMBER OF KNOWN POINTS? 6
X,Y OF POINT 1 ? 20,54
X,Y OF POINT 2 ? 30,90
X,Y OF POINT 3 ? 40,138
X,Y OF POINT 4 ? 50,206
X,Y OF POINT 5 ? 60,292
X,Y OF POINT 6 ? 70,396

              CONSTANT = 41.771428569
 1 DEGREE COEFFICIENT =-1.095714285598
 2 DEGREE COEFFICIENT = 8.78571428E-02


COEFFICIENT OF DETERMINATION (R^2) = .9999279597663
COEFFICIENT OF CORRELATION = .99996397923
STANDARD ERROR OF ESTIMATE = 1.4209319536
```

```
INTERPOLATION:(ENTER 0 TO END PROGRAM)
X =? 55
Y = 247.2750000003

X =? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "NTH-ORDER REGRESSION"
 20  PRINT
 29  REM - SET LIMITS ON DEGREE OF EQUATION TO A(2D+1),R(D+1,D+2),T(D+
    (WHERE D=MAXIMUM DEGREE OF EQUATION)
 30  DIM A(13),R(7,8),T(8)
 40  PRINT "DEGREE OF EQUATION";
 50  INPUT D
 60  PRINT "NUMBER OF KNOWN POINTS";
 70  INPUT N
 80  A(1)=N
 89  REM - ENTER COORDINATES OF DATA POINTS
 90  FOR I=1  TO N
100  PRINT "X,Y OF POINT";I;
110  INPUT X,Y
118  REM - LINES 120-200 POPULATE MATRICES WITH
119  REM - A SYSTEM OF EQUATIONS
120  FOR J=2  TO 2*D+1
130  A(J)=A(J)+X↑(J-1)
140  NEXT J
150  FOR K=1  TO D+1
160  R(K,D+2)=T(K)+Y*X↑(K-1)
170  T(K)=T(K)+Y*X↑(K-1)
180  NEXT K
190  T(D+2)=T(D+2)+Y↑2
200  NEXT I
209  REM - LINES 210-450 SOLVE THE SYSTEM OF EQUATIONS IN THE MATRICES
210  FOR J=1  TO D+1
220  FOR K=1  TO D+1
230  R(J,K)=A(J+K-1)
240  NEXT K
250  NEXT J
260  FOR J=1  TO D+1
270  K=J
280  IF R(K,J)<>0  THEN  320
290  K=K+1
295  IF K<=D+1  THEN  280
300  PRINT "NO UNIQUE SOLUTION"
310  GOTO  790
320  FOR I=1  TO D+2
330  S=R(J,I)
340  R(J,I)=R(K,I)
350  R(K,I)=S
360  NEXT I
370  Z=1/R(J,J)
380  FOR I=1  TO D+2
390  R(J,I)=Z*R(J,I)
400  NEXT I
```

```
410  FOR K=1  TO D+1
420  IF K=J  THEN  470
430  Z=-R(K,J)
440  FOR I=1  TO D+2
450  R(K,I)=R(K,I)+Z*R(J,I)
460  NEXT I
470  NEXT K
480  NEXT J
490  PRINT
495  PRINT "                    CONSTANT =";R(1,D+2)
499  REM - PRINT EQUATION COEFFICIENTS.
500  FOR J=1  TO D
510  PRINT J;"DEGREE COEFFICIENT =";R(J+1,D+2)
520  NEXT J
530  PRINT
539  REM - COMPUTE REGRESSION ANALYSIS
540  P=0
550  FOR J=2  TO D+1
560  P=P+R(J,D+2)*(T(J)-A(J)*T(1)/N)
570  NEXT J
580  Q=T(D+2)-T(1)↑2/N
590  Z=Q-P
600  I=N-D-1
620  PRINT
630  J=P/Q
640  PRINT "COEFFICIENT OF DETERMINATION (R↑2) =";J
650  PRINT "COEFFICIENT OF CORRELATION =";SQR(J)
660  PRINT "STANDARD ERROR OF ESTIMATE =";SQR(Z/I)
670  PRINT
679  REM - COMPUTE Y-COORDINATE FROM ENTERED X -COORDINATE
680  PRINT "INTERPOLATION: (ENTER 0 TO END PROGRAM)"
690  P=R(1,D+2)
700  PRINT "X =";
710  INPUT X
720  IF X=0  THEN  790
730  FOR J=1  TO D
740  P=P+R(J+1,D+2)*X↑J
750  NEXT J
760  PRINT "Y =";P
770  PRINT
780  GOTO  690
790  END
```

Geometric Regression

This program fits a geometric curve to a set of coordinates using the method of least squares. The equation, coefficient of determination, coefficient of correlation and standard error of estimate are printed.

You must provide the x - and y -coordinates of known data points. Once the curve has been fitted you may predict values of y for given values of x.

Example:

The table below gives the pressures of a gas measured at various volumes in an experiment. The relationship between pressure and volume of a gas is expressed by the following formula:

$$PV^K = C$$

where: P = pressure
V = volume
C and K are constants.

This formula can be rewritten in standard geometric form:

$$P = CV^{-K}$$

Note the exponent is negative, which accounts for the negative exponents the program calculates.

Fit a geometric curve to the data and estimate the pressure of 90 cubic inches of the gas.

volume	56.1	60.7	73.2	88.3	120.1	187.5
pressure	57.0	51.0	39.2	30.2	19.6	10.5

```
:RUN
GEOMETRIC REGRESSION

NUMBER OF KNOWN POINTS? 6
X,Y OF POINT 1 ? 56.1,57.0
X,Y OF POINT 2 ? 60.7,51.0
X,Y OF POINT 3 ? 73.2,39.2
X,Y OF POINT 4 ? 88.3,30.2
X,Y OF POINT 5 ? 120.1,19.6
X,Y OF POINT 6 ? 187.5,10.5

F(X) = 16103.68991715 * X↑-1.401550582441

COEFFICIENT OF DETERMINATION (R↑2) = .9999980312731
COEFFICIENT OF CORRELATION = .99999941564
STANDARD ERROR OF ESTIMATE = 7.73614568E-04

INTERPOLATION: (ENTER X=0 TO END PROGRAM)
X =? 90
Y = 29.37349825098

X =? 0

END PROGRAM
```

ROGRAM LISTING

```
 10  PRINT "GEOMETRIC REGRESSION"
 20  PRINT
 30  PRINT "NUMBER OF KNOWN POINTS";
 40  INPUT N
 50  J=0
 60  K=0
 70  L=0
 80  M=0
 90  R2=0
 99  REM - ENTER COORDINATES OF DATA POINTS
100  FOR I=1  TO N
110  PRINT "X,Y OF POINT";I;
120  INPUT X,Y
129  REM - ACCUMULATE INTERMEDIATE VALUES
130  Y=LOG(Y)
140  X=LOG(X)
150  J=J+X
160  K=K+Y
170  L=L+X↑2
180  M=M+Y↑2
190  R2=R2+X*Y
200  NEXT I
209  REM - CALCULATE AND PRINT COEFFICIENTS OF EQUATION
210  B=(N*R2-K*J)/(N*L-J↑2)
220  A=(K-B*J)/N
230  PRINT
240  PRINT "F(X) =";EXP(A);"* X↑";B
249  REM - CALCULATE REGRESSION ANALYSIS
250  J=B*(R2-J*K/N)
260  M=M-K↑2/N
270  K=M-J
280  PRINT
290  R2=J/M
300  PRINT "COEFFICIENT OF DETERMINATION (R↑2) =";R2
310  PRINT "COEFFICIENT OF CORRELATION =";SQR(R2)
320  PRINT "STANDARD ERROR OF ESTIMATE =";SQR(K/(N-2))
330  PRINT
339  REM - ESTIMATE Y-COORDINATE FROM ENTERED X-COORDINATE
340  PRINT "INTERPOLATION:  (ENTER X=0 TO END PROGRAM)"
350  PRINT "X =";
360  INPUT X
370  IF X=0  THEN  410
380  PRINT "Y =";EXP(A)*X↑B
390  PRINT
400  GOTO  350
410  END
```

Exponential Regression

This program finds the coefficients of an equation for an exponential curve. The equation is in the following form:

$$f(x) = ae^{bx}$$

where a and b are the calculated coefficients.

The equation coefficients, coefficient of determination, coefficient of correlation and standard error of estimate are printed.

You must provide the x - and y -coordinates for known data points. Once the curve has been fitted you may predict values of y for given values of x.

Example:

The table below shows the number of bacteria present in a culture at various points in time. Fit an exponential curve to the data and estimate the number of bacteria after 7 hours.

number of hours	0	1	2	3	4	5	6
number of bacteria	25	38	58	89	135	206	315

```
:RUN
EXPONENTIAL REGRESSION

NUMBER OF KNOWN POINTS? 7
X,Y OF POINT 1 ? 0,25
X,Y OF POINT 2 ? 1,38
X,Y OF POINT 3 ? 2,58
X,Y OF POINT 4 ? 3,89
X,Y OF POINT 5 ? 4,135
X,Y OF POINT 6 ? 5,206
X,Y OF POINT 7 ? 6,315

A = 24.96166337346
B = .4223750795699

COEFFICIENT OF DETERMINATION (R↑2) = .9999935513734
COEFFICIENT OF CORRELATION = .99999677568
STANDARD ERROR OF ESTIMATE = 2.53820862E-03

INTERPOLATION: (ENTER X=0 TO END PROGRAM)
X =? 7
Y = 480.0867130787

X =? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "EXPONENTIAL REGRESSION"
 20  PRINT
 30  PRINT "NUMBER OF KNOWN POINTS";
 40  INPUT N
 50  J=0
 60  K=0
 70  L=0
 80  M=0
 90  R2=0
 99  REM - ENTER COORDINATES OF DATA POINTS
100  FOR I=1  TO N
110  PRINT "X,Y OF POINT";I;
120  INPUT X,Y
129  REM - ACCUMULATE INTERMEDIATE VALUES
130  Y=LOG(Y)
140  J=J+X
150  K=K+Y
160  L=L+X↑2
170  M=M+Y↑2
180  R2=R2+X*Y
190  NEXT I
199  REM - CALCULATE AND PRINT COEFFICIENTS OF EQUATION
200  B=(N*R2-K*J)/(N*L-J↑2)
210  A=(K-B*J)/N
220  PRINT
230  PRINT "A =";EXP(A)
240  PRINT "B =";B
249  REM - CALCULATE REGRESSION TABLE VALUES
250  J=B*(R2-J*K/N)
260  M=M-K↑2/N
270  K=M-J
280  PRINT
290  R2=J/M
300  PRINT "COEFFICIENT OF DETERMINATION (R↑2) =";R2
310  PRINT "COEFFICIENT OF CORRELATION =";SQR(R2)
320  PRINT "STANDARD ERROR OF ESTIMATE =";SQR(K/(N-2))
330  PRINT
339  REM - ESTIMATE Y-VALUE FROM ENTERED X-VALUE
340  PRINT "INTERPOLATION: (ENTER X=0 TO END PROGRAM)"
350  PRINT "X =";
360  INPUT X
370  IF X=0  THEN  410
380  PRINT "Y =";EXP(A)*EXP(B*X)
390  PRINT
400  GOTO  350
410  END
```

System Reliability

This program calculates the reliability of an operating system that is subject to wearout and chance failure. You must enter the system's operating time and the wearout time and failure rate of each component.

Example:

Compute the reliability of a computer system operating for 1000 hours with the components shown in the list below.

	wearout (hrs.)	failure
CPU	15,000	.00020
terminal	3,000	.00010
disk	3,000	.00015
printer	1,500	.00015

```
:RUN
SYSTEM RELIABILITY

(TO END PROGRAM ENTER 0)
OPERATING TIME IN HOURS? 1000
NUMBER OF COMPONENTS? 4
COMPONENT 1
  AVERAGE WEAROUT TIME? 15000
  AVERAGE FAILURE RATE? .0002
COMPONENT 2
  AVERAGE WEAROUT TIME? 3000
  AVERAGE FAILURE RATE? .0001
COMPONENT 3
  AVERAGE WEAROUT TIME? 3000
  AVERAGE FAILURE RATE? .00015
COMPONENT 4
  AVERAGE WEAROUT TIME? 1500
  AVERAGE FAILURE RATE? .00015

SYSTEM RELIABILITY = .1353352832367

OPERATING TIME IN HOURS? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "SYSTEM RELIABILITY"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER 0)"
 40  PRINT "OPERATING TIME IN HOURS";
 50  INPUT T
```

```
 59  REM - TEST FOR END OF PROGRAM
 60  IF T=0  THEN  230
 70  PRINT "NUMBER OF COMPONENTS";
 80  INPUT N
 90  Z=0
 99  REM - ENTER DATA FOR EACH COMPONENT
100  FOR I=1  TO N
110  PRINT "COMPONENT";I
120  PRINT "  AVERAGE WEAROUT TIME";
130  INPUT W
140  PRINT "  AVERAGE FAILURE RATE";
150  INPUT F
159  REM - INCLUDE EACH COMPONENT IN RELIABILITY
160  Z=Z+1/W+F
170  NEXT I
180  PRINT
189  REM - CALCULATE RELIABILITY, PRINT
190  Z=EXP(-Z*T)
200  PRINT "SYSTEM RELIABILITY =";Z
210  PRINT
219  REM - RESTART PROGRAM
220  GOTO   40
230  END
```

Average Growth Rate, Future Projections

This program calculates the average growth rate of a company, then projects figures for future years. The growth rate and projections could be computed for any aspect of a company, such as sales, earnings, number of employees, or patronage. You must provide established figures for a past series of years.

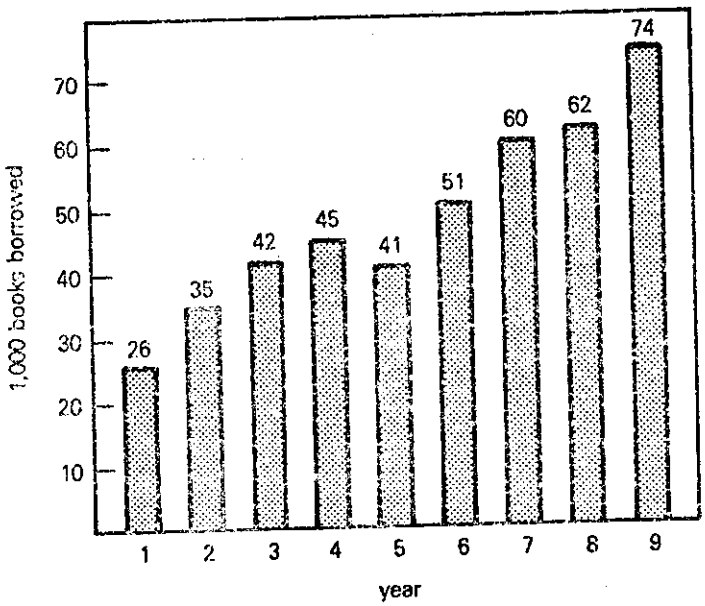
The dimension statement at line 30 limits the number of past figures you may enter. Any need to alter this limit should be done in the following manner:

```
30 DIM S(N)
```

where N = the number of years for which figures are known.

Example:

The borrowing records for Claremount County Library are tabulated in the graph below. What is its average growth rate? How many books can it expect to lend in its tenth and twentieth years of service?




```
:30 DIM S(9)
:RUN
AVERAGE GROWTH RATE, FUTURE PROJECTIONS

NUMBER OF YEARS FIGURES ESTABLISHED? 9
FIGURE: YEAR 1 ? 26
        YEAR 2 ? 35
        YEAR 3 ? 42
        YEAR 4 ? 45
        YEAR 5 ? 41
        YEAR 6 ? 51
        YEAR 7 ? 60
        YEAR 8 ? 62
        YEAR 9 ? 74
AVERAGE GROWTH RATE = 11.88 %
```

```
(ENTER 0 TO END PROGRAM)
PROJECTED SALES FOR YEAR? 10
                                = 81.29
PROJECTED SALES FOR YEAR? 20
                                = 249.88
PROJECTED SALES FOR YEAR? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "AVERAGE GROWTH RATE, FUTURE PROJECTIONS"
 20  PRINT
 29  REM - SET ARRAY S TO NUMBER OF YEARS PAST FIGURES KNOWN
 30  DIM S(20)
 39  REM - STATEMENTS 40 TO 120 REQUEST USER INPUT
 40  PRINT "NUMBER OF YEARS FIGURES ESTABLISHED";
 50  INPUT N
 60  FOR I=1  TO N
 70  IF I>1  THEN  100
 80  PRINT "FIGURE: YEAR";I;
 90  GOTO  110
100  PRINT "         YEAR";I;
110  INPUT S(I)
120  NEXT I
129  REM - INITIALIZE VARIABLES FOR FIRST YEAR
130  T=LOG(S(1))
140  V=0
149  REM - LOOP FOR REMAINING YEARS OF HISTORY
150  FOR I=2  TO N
160  L=LOG(S(I))
170  T=T+L
180  V=V+(I-1)*L
190  NEXT I
199  REM - CALCULATE AVERAGE GROWTH RATE
200  A=6*(2*V/(N-1)-T)/(N)/(N+1)
210  G=EXP(A)-1
219  REM - ROUND OFF, PRINT
220  PRINT "AVERAGE GROWTH RATE =";INT(G*10000+.5)/100;"%"
230  PRINT
239  REM - CALCULATE AVERAGE ANNUAL GROWTH FACTOR
240  S=EXP(T/N-A*(N-1)/2)
250  PRINT "(ENTER 0 TO END PROGRAM)"
259  REM - INPUT YEAR NUMBER
260  PRINT "PROJECTED SALES FOR YEAR";
270  INPUT Y1
279  REM - TEST FOR END OF PROGRAM
280  IF Y1=0  THEN  320
289  REM - CALCULATE PROJECTED SALES FIGURE
290  S1=S*(1+G)↑(Y1-1)
299  REM - ROUND OFF, PRINT                    =";INT(S1*100+.5)/100
300  PRINT "
309  REM - RETURN FOR MORE DATA
310  GOTO  260
320  END
```

Federal Withholding Taxes

This program calculates the amount of federal income and FICA taxes withheld from one's earnings. You must provide employee information as to marital status, the number of exemptions claimed, the amount of taxable pay, and year-to-date taxable pay.

The number of pay periods per year is established at line 80. If your pay period is other than monthly, you must alter this statement to set *N* equal to the number of pay periods per year.

There is a considerable amount of tax information which may change from year to year. The values listed in the data tables at lines 30 and 40 are among those that may need periodic revision. The annual values for single and married persons should be compared each year with those listed in Table 7 of the current IRS Circular E.

The annual FICA rate, the FICA cutoff amount and the annual amount of withholding allowance may also need revision. The values established at lines 50, 60 and 70 should also be compared to those listed in the current IRS circular.

Annual rates and cutoffs are used irrespective of your actual pay period frequency. The program automatically adjusts them to match your pay period.

Examples:

Judy earns $900.00 per month. The payroll clerk is figuring her March paycheck. Judy is single and claims only herself as a dependent. What amounts are withheld from her paycheck?

Dr. Berger has earned $1,408.75 this month. So far this year he has grossed $20,188.72. He is married and claims four dependents. What amounts will be withheld this month for the federal government?

```
:RUN
FEDERAL WITHHOLDING TAXES


MARITAL STATUS (1=SINGLE, 2=MARRIED)? 1
WITHHOLDING TAX EXEMPTIONS? 1
TAXABLE PAY? 900
YTD TAXABLE PAY? 1800
TAXABLE = $ 900
INCOME TAX = $ 128.5
FICA = $ 55.17

MORE DATA (1=YES, 0=NO)? 1

MARITAL STATUS (1=SINGLE, 2=MARRIED)? 2
WITHHOLDING TAX EXEMPTIONS? 4
TAXABLE PAY? 1408.75
YTD TAXABLE PAY? 23750.03
TAXABLE = $ 1408.75
INCOME TAX = $ 152.09
FICA = $ 0

MORE DATA (1=YES, 0=NO)? 0
```

OGRAM LISTING

```
 10  PRINT "FEDERAL WITHHOLDING TAXES"
 20  PRINT
 27  REM - THE FOLLOWING DATA CONTAINS THE 1980 TAX TABLES FROM IRS
 28  REM - CIRCULAR E, PERCENTAGE METHOD, TABLE 7 (ANNUAL PAYROLL)
 29  REM - FOR SINGLE PERSONS
 30  DATA 15,1420,18,3300,21 ,6800,26,10200,30,14200,34,17200,39,22500
 39  REM - FOR MARRIED PERSONS
 40  DATA 15,2400,18,6600,21 ,10900,24,15000,28,19200,32,23600,37,28900
 49  REM - F1=FICA RATE AS DECIMAL
 50  F1=.0613
 59  REM - F2=FICA CUTOFF AMOUNT
 60  F2=25900
 69  REM - W1=AMOUNT OF WITHHOLDING ALLOWANCE (ANNUAL PAYROLL)
 70  W1=1000
 79  REM - N=NUMBER OF PAY PERIODS PER YEAR
 80  N=12
 89  REM - LOAD THE TAX TABLE ARRAYS FROM DATA TABLES
 90  DIM F1(28)
100  FOR I=1  TO 28
110  READ F1(I)
120  NEXT I
130  PRINT
139  REM - STATEMENTS 140 TO 210 REQUEST PERTINENT EMPLOYEE DATA
140  PRINT "MARITAL STATUS (1=SINGLE, 2=MARRIED)";
150  INPUT S
160  PRINT "WITHHOLDING TAX EXEMPTIONS";
170  INPUT W
180  PRINT "TAXABLE PAY";
190  INPUT P
199  REM - Y=TOTAL TAXABLE PAY THIS YEAR, EXCLUDING CURRENT PAYCHECK
200  PRINT "YTD TAXABLE PAY";
210  INPUT Y
219  REM - ANNUALIZE CURRENT TAXABLE PAY, ADJUST FOR EXEMPTIONS
220  G=P*N-W1*W
230  T1=0
239  REM - CALCULATE INCOME TAX
240  FOR I=2  TO 7
250  X=2*I+14*(S-1)-1
260  IF G<=F1(X-1)  THEN  330
270  IF G>F1(X+1)  THEN  300
280  T1=T1+(G-F1(X-1))*F1(X-2)/100
290  GOTO  330
300  T1=T1+(F1(X+1)-F1(X-1))*F1(X-2)/100
310  NEXT I
320  T1=T1+(G-F1(X+1))*F1(X)/100
329  REM - ROUND OFF TO NEAREST CENT
330  T1=INT((T1/N)*100+.5)/100
340  T2=0
349  REM - CALCULATE FICA
350  IF Y>F2  THEN  400
360  IF Y+P>F2  THEN  390
370  T2=INT((P*F1)*100+.5)/100
380  GOTO  400
389  REM - ROUND OFF TO NEAREST CENT
390  T2=INT(((F2-Y)*F1)*100+.5)/100
```

```
399  REM - PRINT RESULTS
400  PRINT "TAXABLE = $";P
410  PRINT "INCOME TAX = $";T1
420  PRINT "FICA = $";T2
430  PRINT
439  REM - RESTART OR END PROGRAM?
440  PRINT "MORE DATA (1=YES, 0=NO)";
450  INPUT S
460  IF S=1  THEN  130
470  END
```

Tax Depreciation Schedule

This program tabulates annual depreciation amounts. You can use the sum of digits method or any declining balance percentage method. You must know the purchase price (initial value), salvage value at the end of the depreciable life, and the life of the item being depreciated. If you are doing declining balance depreciation, you must also know the percentage method.

Examples:

The Miracle Corporation put a new roof on their office building for $27,000.00. They expect to replace it in nine years. What would the annual depreciation amounts be using the sum of digits?

Heavenly Bank built a new home office building for $1.2 million. Run a tax depreciation schedule on the building using 150% declining balance method with a 30 year life. Assume a salvage value of $250,000. You will notice that the depreciation falls below straight line ($31,666.67) per year) at year nine.

```
:RUN
TAX DEPRECIATION SCHEDULE

PURCHASE PRICE? 27000
SALVAGE VALUE? 0
LIFE IN YEARS? 9
ENTER 1 FOR SUM OF DIGITS, 2 FOR DECLINING BALANCE? 1


      SUM OF DIGITS TAX DEPRECIATION
              PRICE $ 27000
         SALVAGE VALUE $ 0
       NET DEPRECIATED $ 27000
              LIFE 9 YEARS

YEAR              DEPRECIATION     BALANCE
 1                 5400             21600
 2                 4800             16800
 3                 4200             12600
 4                 3600             9000
 5                 3000             6000
 6                 2400             3600
 7                 1800             1800
 8                 1200             600
 9                 600              0

MORE DATA?(1=YES,0=NO)? 1

PURCHASE PRICE? 1200000
SALVAGE VALUE? 250000
LIFE IN YEARS? 30
ENTER 1 FOR SUM OF DIGITS, 2 FOR DECLINING BALANCE? 2
METHOD IN %? 150
```

```
      DECLINING BALANCE TAX DEPRECIATION
            PRICE $ 1200000
         SALVAGE VALUE $ 250000
       NET DEPRECIATED $ 950000
            LIFE 30 YEARS
            METHOD  150 %

YEAR              DEPRECIATION     BALANCE
 1                 47500            902500
 2                 45125            857375
 3                 42868.75         814506.25
 4                 40725.31         773780.94
 5                 38689.05         735091.89
 6                 36754.59         698337.3
 7                 34916.87         663420.43
 8                 33171.02         630249.41
 9                 31512.47         598736.94
 10                29936.85         568800.09
 11                28440            540360.09
 12                27018            513342.09
 13                25667.1          487674.99
 14                24383.75         463291.24
 15                23164.56         440126.68
 16                22006.33         418120.35
 17                20906.02         397214.33
 18                19860.72         377353.61
 19                18867.68         358485.93
 20                17924.3          340561.63
 21                17028.08         323533.55
 22                16176.68         307356.87
 23                15367.84         291989.03
 24                14599.45         277389.58
 25                13869.48         263520.1
 26                13176.01         250344.09
 27                12517.2          237826.89
 28                11891.34         225935.55
 29                11296.78         214638.77
 30                10731.94         203906.83

MORE DATA?(1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  10  PRINT "TAX DEPRECIATION SCHEDULE"
  20  PRINT
  29  REM - ENTER INITIAL VALUE AND ROUND OFF TO NEAREST CENT
  30  PRINT "PURCHASE PRICE";
  40  INPUT V
  50  V=INT(V*100+.5)/100
  59  REM - ENTER END VALUE AND ROUND OFF TO NEAREST CENT
  60  PRINT "SALVAGE VALUE";
  70  INPUT S
  80  S=INT(S*100+.5)/100
  89  REM - COMPUTE AMOUNT TO DEPRECIATE
```

```
 90  D=V-S
 99  REM - ENTER LENGTH OF DEPRECIATION
100  PRINT "LIFE IN YEARS";
110  INPUT Y
119  REM - CHOOSE DEPRECIATION METHOD
120  PRINT "ENTER 1 FOR SUM OF DIGITS, 2 FOR DECLINING BALANCE";
130  INPUT X
140  IF X=2  THEN  450
150  IF X<>1  THEN  120
158  REM - BY SUM OF DIGITS METHOD
159  REM - R1 IS THE CUMULATIVE AMOUNT DEPRECIATED
160  R1=0
169  REM - N IS THE PRINTED LINE COUNTER
170  N=66
180  PRINT
190  PRINT
200  FOR I=1  TO Y
209  REM - TEST FOR FULL PAGE
210  IF N<55  THEN  330
219  REM - FULL PAGE; SPACE TO TOP OF NEXT PAGE AND PRINT HEADINGS
220  FOR I1=N  TO 66
230  PRINT
240  NEXT I1
250  N=7
260  PRINT "      SUM OF DIGITS TAX DEPRECIATION"
270  PRINT "                PRICE $";V
280  PRINT "          SALVAGE VALUE $";S
290  PRINT "       NET DEPRECIATED $";V-S
300  PRINT "              LIFE";Y;"YEARS"
310  PRINT
320  PRINT "YEAR","DEPRECIATION","BALANCE"
329  REM - COMPUTE DEPRECIATION AND ROUND OFF TO NEAREST CENT
330  R=2*D*(Y-I+1)/((Y+1)*Y)
340  R=INT(R*100+.5)/100
349  REM - ACCUMULATE DEPRECIATION
350  R1=R1+R
359  REM - COMPUTE BALANCE TO DEPRECIATE
360  B=D-R1
369  REM - TEST FOR COMPLETE DEPRECIATION
370  IF B>=0  THEN  410
380  R1=R1+B
390  R=R+B
400  B=0
410  PRINT I,R,B
420  N=N+1
430  NEXT I
440  GOTO  700
448  REM - BY DECLINING BALANCE METHOD
449  REM - ENTER DECLINING BALANCE PERCENT
450  PRINT "METHOD IN %";
460  INPUT M
469  REM - CONVERT PERCENT TO DECIMAL
470  M=M/100
479  REM - N  COUNTS THE LINES PRINTED ON EACH PAGE
480  N=66
489  REM - R IS THE AMOUNT LEFT TO DEPRECIATE
490  R=D
```

```
500  PRINT
510  FOR I=1  TO Y
519  REM - TEST FOR A FULL PRINTED PAGE
520  IF N<55  THEN  650
529  REM - FULL PAGE; SPACE TO TOP OF NEXT PAGE AND PRINT HEADINGS
530  FOR I1=N  TO 66
540  PRINT
550  NEXT I1
560  N=8
570  PRINT "      DECLINING BALANCE TAX DEPRECIATION"
580  PRINT "            PRICE $";V
590  PRINT "         SALVAGE VALUE $";S
600  PRINT "       NET DEPRECIATED $";D
610  PRINT "            LIFE";Y;"YEARS"
620  PRINT "            METHOD ";M*100;"%"
630  PRINT
640  PRINT "YEAR","DEPRECIATION","BALANCE"
649  REM - COMPUTE DEPRECIATION AND ROUND OFF TO THE NEAREST CENT
650  R1=INT((R*M/Y)*100+.5)/100
659  REM - ACCUMULATE REMAINING BALANCE
660  R=R-R1
670  PRINT I,R1,R
680  N=N+1
690  NEXT I
700  PRINT
709  REM - RESTART OR END PROGRAM?
710  PRINT "MORE DATA?(1=YES,0=NO)";
720  INPUT X
730  IF X=1  THEN  20
740  END
```

Check Writer

This program prints a check. You must provide the date, amount and payee of the check. The program translates the date and amount to words and prints providing spacing within the check.

You should regard the program listed below as a sample of a check-writing program. Very few checks will conform exactly to the spacing provided in this program. The method of translating words from numbers is generally applicable. Spacing should be altered to conform to your own check format.

When the program asks the question READY TO PRINT CHECK? it is prompting you to insert a blank check in your printing device. The check should be set one line above the line on which the date is to be printed.

Once the check is set up, key RETURN (no other entry is required) and the check will be printed.

Example:

Among the checks that Miracle Corporation must write are one to Osborne & Associates for $4975.89 and one to Freida Alexander for $103.75. Print the checks using the computer.

```
:RUN
CHECK WRITER

DATE (MMDDYY)? 308'77
--(TO END PROGRAM ENTER 'END')--
FIRST NAME OF PAYEE? OSBORNE &
LAST NAME OF PAYEE? ASSOCIATES
AMOUNT OF CHECK? 4975.89
READY TO PRINT CHECK?
```

NO. 328

HEAVENLY BANK
EMERYVILLE OFFICE
4120 ASHBY AVENUE
EMERYVILLE, CA 94601

MARCH 8 19 77
$**4975.89
AMOUNT $ 4975.89

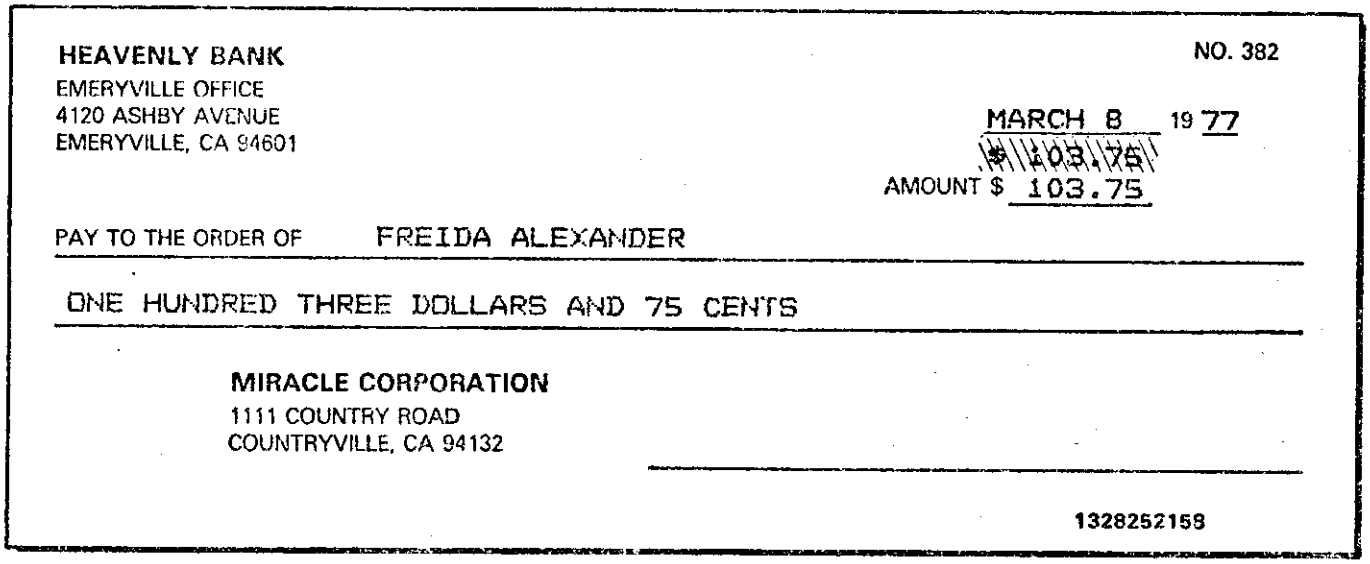
PAY TO THE ORDER OF OSBORNE & ASSOCIATES

FOUR THOUSAND NINE HUNDRED SEVENTY-FIVE DOLLARS AND 89 CENTS

MIRACLE CORPORATION
1111 COUNTRY ROAD
COUNTRYVILLE, CA 94132

1328252158

```
FIRST NAME OF PAYEE? FREIDA
LAST NAME OF PAYEE? ALEXANDER
AMOUNT OF CHECK? 103.75
READY TO PRINT CHECK?
```



HEAVENLY BANK
EMERYVILLE OFFICE
4120 ASHBY AVENUE
EMERYVILLE, CA 94601

NO. 382

MARCH 8 19 77

$ 103.75

AMOUNT $ 103.75

PAY TO THE ORDER OF FREIDA ALEXANDER

ONE HUNDRED THREE DOLLARS AND 75 CENTS

MIRACLE CORPORATION
1111 COUNTRY ROAD
COUNTRYVILLE, CA 94132

1328252158

```
FIRST NAME OF PAYEE? END

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "CHECK WRITER"
 20  PRINT
 30  DATA "ONE","TWO","THREE","FOUR","FIVE","SIX","SEVEN","EIGHT","NINE"
 40  DATA "TEN","ELEVEN","TWELVE","THIRTEEN","FOURTEEN","FIFTEEN","SIXTEE
     N"
 50  DATA "SEVENTEEN","EIGHTEEN","NINETEEN","TWENTY","THIRTY","FORTY"
 60  DATA "FIFTY","SIXTY","SEVENTY","EIGHTY","NINETY"
 70  DATA "JANUARY","FEBRUARY","MARCH","APRIL","MAY","JUNE","JULY"
 80  DATA "AUGUST","SEPTEMBER","OCTOBER","NOVEMBER","DECEMBER"
 89  REM - ENTER DATE WITHOUT COMMAS; DAY AND YEAR MUST CONTAIN TWO DIGIT
     S
 90  PRINT "DATE (MMDDYY)";
100  INPUT D
110  PRINT "--(TO END PROGRAM ENTER 'END')--"
120  PRINT "FIRST NAME OF PAYEE";
130  INPUT F$
139  REM - END PROGRAM?
140  IF F$="END"  THEN  790
150  PRINT "LAST NAME OF PAYEE";
160  INPUT L$
170  PRINT "AMOUNT OF CHECK";
180  INPUT A
189  REM - INSERT BLANK CHECK IN PRINTING DEVICE, KEY RETURN WHEN READY
190  PRINT "READY TO PRINT CHECK";
200  INPUT X
209  REM - BREAK ENTERED DATE NUMBER INTO MONTH, DAY, YEAR FIGURES
210  D1=INT(D/10000)
```

```
220  D2=INT((D-D1*10000)/100)
230  D3=INT(D-(D1*100+D2)*100)
239  REM - GO TO CORRECT MONTH IN DATA TASLE
240  RESTORE 27+D1
250  READ X0$
259  REM - PRINT DATE
260  PRINT ,,,X0$;D2;"  ";D3
269  REM - PRINT AMOUNT TWICE; FIRST TIME FOR SHADED BOX
270  PRINT ,,,"$";A
280  PRINT ,,," ";A
290  PRINT
300  PRINT ,F$;" ";L$
310  PRINT
319  REM - AMOUNT OF CHECK LEGITIMATE?
320  IF A<=0  THEN  770
330  A1=A
339  REM - AMOUNT IN THE THOUSANDS?
340  N1=INT(A1/1E3)
349  REM - CAN'T PRINT FOR AMOUNT OVER $99999.99
350  IF N1>99 THEN  770
360  IF N1=0  THEN  390
370  GOSUB  640
380  PRINT "THOUSAND ";
390  A1=A1-N1*1E3
399  REM - AMOUNT IN THE HUNDREDS?
400  N1=INT(A1/100)
410  IF N1=0  THEN  440
420  GOSUB  640
430  PRINT "HUNDRED ";
440  A1=A1-N1*100
449  REM - AMOUNT IN THE ONES OR TENS?
450  N1=INT(A1)
460  IF N1>0  THEN  490
470  IF A>=1  THEN  500
480  GOTO  510
490  GOSUB  640
500  PRINT "DOLLARS ";
510  A1=A1-N1
519  REM - ANY CENTS?
520  IF A1<.01  THEN  600
529  REM - IF AMOUNT IS CENTS ONLY DON'T PRINT 'AND'
530  IF A<1  THEN  550
540  PRINT "AND";
550  A1=A1*100
559  REM - CENTS ARE PRINTED IN NUMERIC FORM
560  PRINT A1;"CENTS"
569  REM - SPACE OFF OF CHECK
570  PRINT
580  PRINT
590  PRINT
600  PRINT
610  PRINT
620  PRINT
629  REM - RESTART PROGRAM
630  GOTO  120
639  REM - SUBROUTINE TO GET WORDS FOR NUMBERS
640  IF N1<21  THEN  730
```

```
650  RESTORE (N1-20)/10+20
660  READ X0$
670  PRINT X0$;
680  A3=N1-INT(N1/10)*10
690  IF A3=0  THEN  760
700  PRINT "-";
710  RESTORE A3
720  GOTO  740
730  RESTORE N1
740  READ X0$
750  PRINT X0$;" ";
759  REM - END OF SUBROUTINE
760  RETURN
770  PRINT ,"*****VOID*****"
780  GOTO  570
790  END
```

Recipe Cost

This program calculates the cost and the cost per serving of a single recipe. For each ingredient you must provide the purchase price, the amount purchased, the amount used in the recipe, and the number of recipe units per purchase unit.

Example:

Listed below is a recipe for strawberry shortcake. Calculate the cost of the recipe and the cost per serving. What would the cost per serving be if one cake serves 12? The conversion factors and price per ingredient are supplied.

Strawberry Shortcake — 8 servings

3 c. flour	2.5 c./lb.	$1.59	5 lb.
3¼ tsp. baking powder	15 tsp./oz.	.43	4 oz.
¼ c. sugar	2 c./lb.	1.24	5 lb.
1¼ tsp. salt	6 tsp./oz.	.29	1 lb.
½ c. butter	2 c./lb.	1.49	1 lb.
1 egg	12/doz.	.75	1 doz.
⅔ c. milk	4 c./qt.	.40	1 qt.
3 pts strawberries	——	.49	1 pt.
½ pt. whipping cream	——	.59	½ pt.

```
:RUN
RECIPE COST

NUMBER OF INGREDIENTS? 9
INGREDIENT 1 :
  COST FOR BULK UNIT IN STORE? 1.59
  NUMBER OF UNITS IN BULK? 5
  NUMBER OF RECIPE UNITS PER BULK UNIT? 2.5
  NUMBER OF RECIPE UNITS CALLED FOR? 3
INGREDIENT 2 :
  COST FOR BULK UNIT IN STORE? .43
  NUMBER OF UNITS IN BULK? 4
  NUMBER OF RECIPE UNITS PER BULK UNIT? 15
  NUMBER OF RECIPE UNITS CALLED FOR? 3.25
INGREDIENT 3 :
  COST FOR BULK UNIT IN STORE? 1.24
  NUMBER OF UNITS IN BULK? 5
  NUMBER OF RECIPE UNITS PER BULK UNIT? 2
  NUMBER OF RECIPE UNITS CALLED FOR? .25
INGREDIENT 4 :
  COST FOR BULK UNIT IN STORE? .29
  NUMBER OF UNITS IN BULK? 1
  NUMBER OF RECIPE UNITS PER BULK UNIT? 96
  NUMBER OF RECIPE UNITS CALLED FOR? 1.25
INGREDIENT 5 :
  COST FOR BULK UNIT IN STORE? 1.49
  NUMBER OF UNITS IN BULK? 1
  NUMBER OF RECIPE UNITS PER BULK UNIT? 2
  NUMBER OF RECIPE UNITS CALLED FOR? .5
```

```
INGREDIENT 6 :
  COST FOR BULK UNIT IN STORE? .75
  NUMBER OF UNITS IN BULK? 1
  NUMBER OF RECIPE UNITS PER BULK UNIT? 12
  NUMBER OF RECIPE UNITS CALLED FOR? 1
INGREDIENT 7 :
  COST FOR BULK UNIT IN STORE? .40
  NUMBER OF UNITS IN BULK? 1
  NUMBER OF RECIPE UNITS PER BULK UNIT? 4
  NUMBER OF RECIPE UNITS CALLED FOR? .6666667
INGREDIENT 8 :
  COST FOR BULK UNIT IN STORE? .49
  NUMBER OF UNITS IN BULK? 1
  NUMBER OF RECIPE UNITS PER BULK UNIT? 1
  NUMBER OF RECIPE UNITS CALLED FOR? 3
INGREDIENT 9 :
  COST FOR BULK UNIT IN STORE? .59
  NUMBER OF UNITS IN BULK? 1
  NUMBER OF RECIPE UNITS PER BULK UNIT? 1
  NUMBER OF RECIPE UNITS CALLED FOR? 1
NUMBER OF SERVINGS? 8

TOTAL COST FOR 1 RECIPE = $ 3
COST PER SERVING = $ .38

CHANGE NUMBER OF SERVINGS (1=YES,0=NO)? 1
NUMBER OF SERVINGS? 12

TOTAL COST FOR 1 RECIPE = $ 3
COST PER SERVING = $ .25

CHANGE NUMBER OF SERVINGS (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  10  PRINT "RECIPE COST"
  20  PRINT
  29  REM - STATEMENTS 30 TO 180 REQUEST USER INPUT
  30  PRINT "NUMBER OF INGREDIENTS";
  40  INPUT N
  49  REM - LOOP TO REQUEST DATA FOR EACH INGREDIENT
  50  FOR I=1  TO N
  60  PRINT "INGREDIENT";I;":"
  70  PRINT "  COST FOR BULK UNIT IN STORE";
  80  INPUT C
  90  PRINT "  NUMBER OF UNITS IN BULK";
 100  INPUT U
 110  PRINT "  NUMBER OF RECIPE UNITS PER BULK UNIT";
 120  INPUT F
 130  PRINT "  NUMBER OF RECIPE UNITS CALLED FOR";
 140  INPUT R
 149  REM - SUM COST OF EACH INGREDIENT PER AMOUNT USED
 150  P=P+C/U/F*R
 160  NEXT I
```

```
170   PRINT "NUMBER OF SERVINGS";
180   INPUT S
190   PRINT
199   REM - ROUND OFF COSTS TO NEAREST CENT, PRINT RESULTS
200   PRINT "TOTAL COST FOR 1 RECIPE = $";INT(P*100+.5)/100
210   PRINT "COST PER SERVING = $";INT(P/S*100+.5)/100
220   PRINT
229   REM - CALCULATE ALTERNATIVE PRICE PER SERVING?
230   PRINT "CHANGE NUMBER OF SERVINGS (1=YES,0=NO)";
240   INPUT N
250   IF N=1  THEN  170
260   END
```

OPTION

As you become familiar with the operation of this program you may wish to shorten it by entering the information required for each ingredient on one line. The program changes necessary are listed following the example below.

Example:

Calculate the cost per serving of Strawberry Shortcake in the previous example when it is served without cream.

```
:RUN
RECIPE COST

NUMBER OF INGREDIENTS? 8
INGREDIENT 1 ? 1.50,5,2.5,3
INGREDIENT 2 ? .43,4,15,3.25
INGREDIENT 3 ? 1.24,5,2,.25
INGREDIENT 4 ? .29,1,96,1.25
INGREDIENT 5 ? 1.49,1,2,.5
INGREDIENT 6 ? .75,1,12,1
INGREDIENT 7 ? .40,1,4,.6666667
INGREDIENT 8 ? .49,1,1,3
NUMBER OF SERVINGS? 8

TOTAL COST FOR 1 RECIPE = $ 2.41
COST PER SERVING = $ .3

CHANGE NUMBER OF SERVINGS (1=YES,0=NO)? 1
NUMBER OF SERVINGS? 12

TOTAL COST FOR 1 RECIPE = $ 2.41
COST PER SERVING = $ .2

CHANGE NUMBER OF SERVINGS (1=YES,0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  1  REM - OPTION 55-70
 10  PRINT "RECIPE COST"
  :
 50  FOR I=1  TO N
 55  REM - ENTER C,U,F,R
 56  REM - WHERE C=COST FOR BULK UNIT
 57  REM -       U=NUMBER UNITS IN BULK UNIT
 58  REM -       F=RECIPE UNITS PER BULK UNIT
 59  REM -       R=NUMBER RECIPE UNITS CALLED FOR
 60  PRINT "INGREDIENT";I;
 70  INPUT C,U,F,R
149  REM - SUM COST OF EACH INGREDIENT PER AMOUNT USED
  :
260  END
```

Survey Check (Map Check)

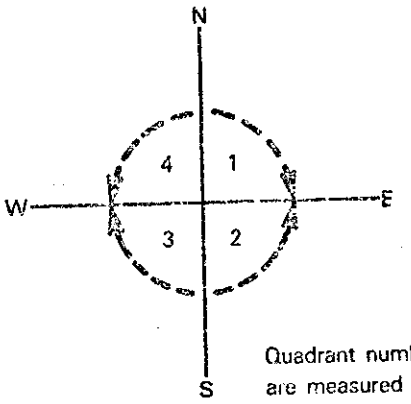
Courtesy: Robert Irving
Northridge,
California

This program calculates the error of closure and area of a plot for which a traverse of the perimeter is available. The program will also calculate how far North and East the end of an open traverse is from its origin (the Northing and Easting). The local coordinates of the origin can be entered for an open traverse. Negative values of Northing and Easting are South and West, respectively, of the 0,0 origin of the survey.

The individual legs of the traverse may be either straight lines or arcs of circles. To compute the traverse, you must have the bearing and length of each straight leg. You also need the radius, bearing of chord, and length of chord (or radius, arc measure, and bearing of a tangent) for each curved leg.

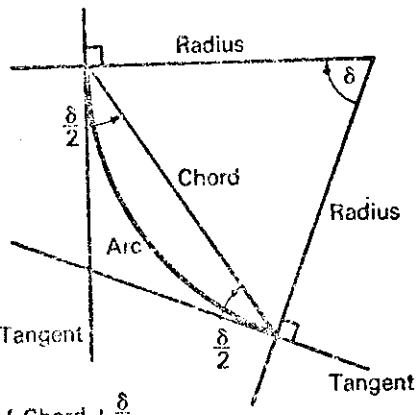
For a closed survey, pick any intersection of legs as a starting point, and number the lines and arcs, starting with one, in a *clockwise* direction around the perimeter. If any arc is 180 degrees or more, it must be broken into smaller arcs, each less than 180 degrees.

By convention, surveyors measure bearings East and West of North and South, as shown in the following figure. This convention was established in the days before computers, so that trigonometric functions could be easily looked up in tables not exceeding 90 degrees. For each leg, you must enter the quadrant number and the degrees, minutes and seconds East or West of the North-South axis. The program will indicate the direction of the leg (e.g., SW), and will convert the quadrant, degrees, etc. to an azimuth angle. Azimuth is measured clockwise from North to 360 degrees.



Quadrant numbering scheme. Note angles are measured from the North-South line.

A curved leg, or arc, is defined by two auxiliary legs, each of which is a radius of the arc. The bearing of the first auxiliary leg is the direction of the radius from the first encountered end of the arc to the center of the arc. You can compute this bearing from the bearing of the arc's tangent at that point, since the radius is perpendicular to the tangent. The survey may show the bearing of the tangent. If not, you can compute it by adding one half the angular extent of the arc to the bearing of the arc's chord, as shown in the next figure.



Bearing of Tangent = Bearing of Chord + $\frac{\delta}{2}$

The bearing of the second radius is from the center of the arc to the other end, and the distance is entered as a *negative* number to signal to the computer that this and the prior leg are not perimeter legs, but auxiliary legs of an arc.

The program asks you for the bearing and distance of each leg by number. Legs are entered in sets of ten (or less). Following the last entry in a set, you can correct any leg in the set. You must enter both auxiliary legs of an arc in the same set. You can enter a bearing of zero to end one set, and then enter more legs on the next set.

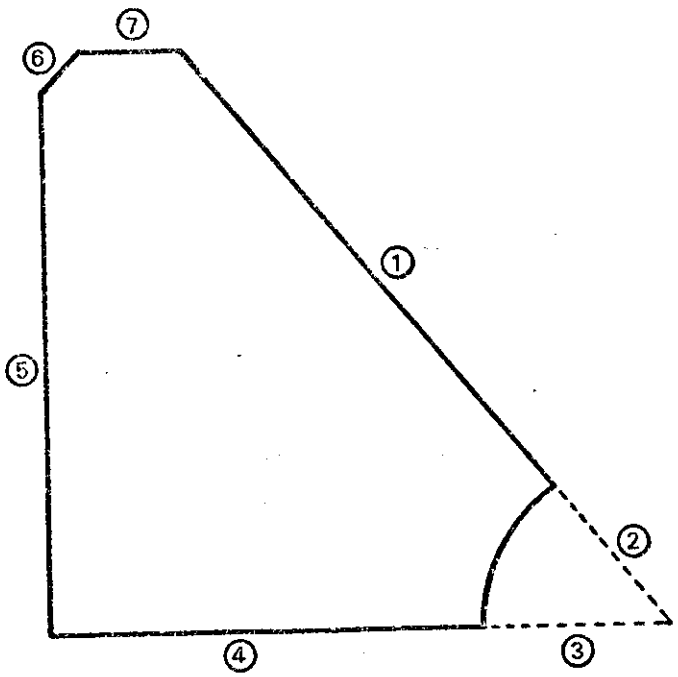
When you have corrected a set, a traverse table is printed for the set. This includes each leg number, direction, azimuth angle and distance, and incremental and cumulative Northing and Easting. The cumulative Northing and Easting after the last leg on a closed survey gives the error of closure. Arc angle, radius, sector area, chord length, and tangent length are printed between the two auxiliary legs of each curved leg.

Following the printout of the last leg of a closed survey, the area of the plot will be printed, both in square feet and in acres. The area computed is very accurate provided two conditions are met:

1) the error of closure is small (0.01 feet is usual for a house lot), and
2) the area is sufficiently small that curvature of the earth does not become significant. Surveys covering several tens of miles have to account for this latter factor.

Example:

The figure below illustrates the boundaries of a lot with one curved side. The leg numbers are circled. Bearings and distances are shown for each leg. Find the error of closure and lot area.



① S39°0"E
149.83
② S39°0"E
50.00
③ N85°23'53"W
50.00
④ N85°23'53"W
114.32
⑤ N1°5'0"E
132.78
⑥ N46°0'0"E
14.00
⑦ S89°0'0"E
25.46

```
:RUN
MAP CHECK - SURVEY CLOSURE & AREA
OPEN (1) OR CLOSED (0) SURVEY? 0
NEXT SET OF LEGS:
LEG NO.  1 :QUADRANT,DEGREES,MINUTES,SECONDS? 2,39,0,0
DISTANCE (NEGATIVE IF OUTWARD RADIUS)? 149.83
LEG NO.  2 :QUADRANT,DEGREES,MINUTES,SECONDS? 2,39,0,0
DISTANCE (NEGATIVE IF OUTWARD RADIUS)? 50
LEG NO.  3 :QUADRANT,DEGREES,MINUTES,SECONDS? 4,85,23,53
DISTANCE (NEGATIVE IF OUTWARD RADIUS)? -50
LEG NO.  4 :QUADRANT,DEGREES,MINUTES,SECONDS? 4,85,23,53
DISTANCE (NEGATIVE IF OUTWARD RADIUS)? 114.32
LEG NO.  5 :QUADRANT,DEGREES,MINUTES,SECONDS? 1,1,5,0
DISTANCE (NEGATIVE IF OUTWARD RADIUS)? 132.78
LEG NO.  6 :QUADRANT,DEGREES,MINUTES,SECONDS? 1,46,0,0
DISTANCE (NEGATIVE IF OUTWARD RADIUS)? 14
LEG NO.  7 :QUADRANT,DEGREES,MINUTES,SECONDS? 2,89,0,0
DISTANCE (NEGATIVE IF OUTWARD RADIUS)? 25.46
LEG NO.  8 :QUADRANT,DEGREES,MINUTES,SECONDS? 0,0,0,0
CORRECT WHICH LEG IN THIS SET (0=NO MORE CHANGES)? 0

LEG/DIR.  AZIMUTH/DIST.                DEL N/DEL E    NORTHING/EASTING
                                                        0 / 0

 1 /SE   141   0   0 / 149.83        -116.44 / 94.291 -116.44 / 94.291

 2 /SE   141   0   0 / 50            -38.857 / 31.466 -155.297 / 125.757

   ARC:   46   23   53 R= 50 A= 2024.497 C= 39.393 T= 21.429

 3 /NW   274   36   7 / 50            4.012 /-49.839 -151.285 / 75.918

 4 /NW   274   36   7 / 114.32        9.172 /-113.951 -142.113 /-36.033

 5 /NE   1   4   60 / 132.78         132.756 / 2.51 -9.357 /-35.523

 6 /NE   46   0   0 / 14             9.725 / 10.071   .368 /-25.452

 7 /SE   91   0   0 / 25.46          -.444 / 25.456 -7.60E-02 / 4.0E-03

ANY MORE LEGS (1=YES, 0=NO)? 0
PLOT AREA IS  13347.693 SQ. FT.

PLOT AREA IS  .30642064 ACRES

STOP
```

PROGRAM LISTING

```
  3  REM - MAP CHECK & AREA OF PLOT
  4  REM - FOR CLOSED SURVEY FOLLOW TRAVERSE CLOCKWISE
  5  REM - KEEP PLOT TO RIGHT OF EACH PERIMETER LEG
  6  REM - COMPUTE AUXILIARY LEGS AS RADII AT EACH
  7  REM - END OF ARC.  ARC < 180 DEGREES
  8  REM
  9  REM - VALUE OF PI
 10  P1=3.141592654
 14  REM - K0 = NO. OF LEGS PER SET
 15  K0=10
 20  DIM B(10),L(10)
 29  REM - C0$ CONTAINS 'CLEAR SCREEN' CHARACTER
 30  C0$=HEX(03)
 39  REM - FNR(X) ROUNDS X TO 3 DECIMAL PLACES
 40  DEFFNR(X)=INT(X*1000+.5)/1000
 49  REM - R IS CONVERSION FACTOR FOR DEGREES TO RADIANS
 50  R=1.745329251E-2
 60  PRINT C0$;"MAP CHECK - SURVEY CLOSURE & AREA"
 70  PRINT "OPEN (1) OR CLOSED (0) SURVEY";
 80  INPUT F
 90  IF F=0  THEN  120
100  PRINT "ORIGIN: NORTHING,EASTING";
110  INPUT N,E
120  PRINT C0$;"NEXT SET OF LEGS:"
125  G=H
130  FOR  K=1  TO K0
139  REM - INPUT BEARING AND DISTANCE FOR NEXT LEG
140  GOSUB 2000
149  REM - IF BEARING IS 0, END INPUT FOR THIS SET
150  IF Q=0  THEN  170
155  G=G+1
160  GOTO  240
169  REM - ZERO UNUSED LEGS IN THIS SET
170  IF K=K0  THEN  230
180  FOR J=K+1  TO K0
190  B(J)=0
200  L(J)=0
210  NEXT J
230  K=K0
240  NEXT K
260  PRINT "CORRECT WHICH LEG IN THIS SET (0=NO MORE CHANGES)";
270  INPUT K
279  REM - NO CHANGES IF 0 INPUT
280  IF K=0  THEN  310
285  K=K-H
290  GOSUB 2000
300  GOTO  260
309  REM - COMPUTE VALUES AND PRINT TRAVERSE TABLE
310  PRINT
315  PRINT "LEG/DIR.  AZIMUTH/DIST.";
320  PRINT "            DEL N/DEL E   NORTHING/EASTING"
330  PRINT ,,,FNR(N);"/";FNR(E)
340  PRINT
350  FOR K=1  TO K0
360  L1=L(K)
```

```
369  REM - CHECK FOR ARC
370  IF L1<0  THEN 1100
380  IF L1=0  THEN  900
388  REM - COMPUTE NORTHING/EASTING INCREMENT (CONVERT
389  REM - BEARINGS FROM DEGREES TO RADIANS)
390  L=L(K)*COS(B(K)*R)
400  D=L(K)*SIN(B(K)*R)
410  N=N+L
420  E=E+D
429  REM - INCREMENT AREA
430  A=A-E*L+N*D
440  PRINT H+K;"/";
449  REM - FROM BEARING, DETERMINE DIRECTION
450  IF B(K)=0  THEN  470
460  GOTO  490
470  PRINT "N";
480  GOTO  830
490  IF B(K)<90  THEN  510
500  GOTO  530
510  PRINT "NE";
520  GOTO  830
530  IF B(K)=90  THEN  550
540  GOTO  570
550  PRINT "E ";
560  GOTO  830
570  IF B(K)<180  THEN  590
580  GOTO  610
590  PRINT "SE";
600  GOTO  830
610  IF B(K)=180  THEN  630
620  GOTO  650
630  PRINT "S ";
640  GOTO  830
650  IF B(K)<270  THEN  670
660  GOTO  690
670  PRINT "SW";
680  GOTO  830
690  IF B(K)=270  THEN  710
700  GOTO  730
710  PRINT "W ";
720  GOTO  830
730  IF B(K)<360  THEN  750
740  GOTO  770
750  PRINT "NW";
760  GOTO  830
770  IF B(K)=360  THEN  790
780  GOTO  810
790  PRINT "N ";
800  GOTO  830
810  B(K)=B(K)-360
820  GOTO  450
829  REM - BREAK BEARING INTO DEGREES, MINUTES, SECONDS
830  D1=INT(B(K))
840  M1=(B(K)-D1)*60
850  M=INT(M1)
860  S=INT((M1-M)*60+.5)
870  PRINT "  ";D1;M;S;"/";FNR(L(K)),FNR(L);
```

```
 880  PRINT "/";FNR(D);FNR(N);"/";FNR(E)
 885  PRINT
 890  L(K)=L1
 900  NEXT K
 910  H=G
 920  PRINT "ANY MORE LEGS (1=YES, 0=NO)";
 930  INPUT U
 940  IF U<>0  THEN  120
 949  REM - NO AREA FOR OPEN SURVEY
 950  IF F<>0  THEN 1000
 960  A=ABS(A/2)
 970  PRINT "PLOT AREA IS ";FNR(A);"SQ. FT."
 980  PRINT
 990  PRINT "PLOT AREA IS ";INT(A/43560*1E8+.5)/1E8;"ACRES"
1000  STOP
1099  REM - CALCULATE CURVED LEG AND PRINT ON TRAVERSE TABLE
1100  C=ABS(B(K)-B(K-1))
1110  C=ABS(180-C)
1120  D=-L1
1130  L(K)=D
1140  A1=C/180*P1*D*D
1150  C1=2*D*SIN(C/2*R)
1160  T=D*TAN(C/2*R)
1170  B9=B(K)-B(K-1)
1180  IF B9<-180  THEN 1230
1190  IF B9>180  THEN 1210
1200  IF B9>0  THEN  1230
1210  A=A+A1
1220  GOTO 1240
1230  A=A-A1
1240  D1=INT(C)
1250  M1=(C-D1)*60
1260  M=INT(M1)
1270  S=INT((M1-M)*60+.5)
1280  PRINT "   ARC: ";D1;M;S;"R=";FNR(D);"A=";FNR(A1);"C=";
1290  PRINT FNR(C1);"T=";FNR(T)
1300  PRINT
1320  GOTO  390
1999  REM - INPUT DATA FOR ONE LEG
2000  B(K)=0
2010  L(K)=0
2020  PRINT "LEG NO. ";H+K;":QUADRANT,DEGREES,MINUTES,SECONDS";
2030  INPUT Q,D,M,S
2040  IF Q=0  THEN 2270
2050  IF Q>4  THEN 2020
2060  IF Q<0  THEN 2020
2070  IF D<0  THEN 2020
2080  IF M<0  THEN 2020
2090  IF S<0  THEN 2020
2100  B(K)=D+(M+S/60)/60
2110  IF B(K)>90  THEN  2020
2120  IF Q=1  THEN 2220
2130  IF Q=2  THEN 2150
2140  GOTO 2170
2150  B(K)=180-B(K)
2160  GOTO 2220
2170  IF Q=3  THEN 2190
```

```
2180  GOTO 2210
2190  B(K)=180+B(K)
2200  GOTO 2220
2210  IF Q<>4  THEN 2220
2215  B(K)=360-B(K)
2220  PRINT "DISTANCE (NEGATIVE IF OUTWARD RADIUS)";
2230  INPUT L(K)
2240  IF L(K)>0  THEN 2270
2250  IF ABS(L(K))<>ABS(L(K-1))  THEN 2220
2270  RETURN
9999  END
```

Day of the Week

This program calculates the day of the week that a given date falls on. It will figure, for example, that December 25, 1980 will be a Thursday.

You must enter the date in numeric form and in the order of month, day, year. September 12, 1975 will be entered as `9,12,1975`, making certain that commas, not slashes or dashes, separate the figures.

Examples:

Cindy's birthdate is March 4, 1953. On what day was she born?

Uncle Lon has an appointment on September 30, 1977. What day is that on?

```
:RUN
DAY OF THE WEEK

(ENTER 0,0,0 TO END PROGRAM)
MONTH, DAY, YEAR? 3,4,1953
WEDNESDAY

MONTH, DAY, YEAR? 9,30,1977
FRIDAY

MONTH, DAY, YEAR? 0,0,0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "DAY OF THE WEEK"
 20  PRINT
 29  REM - REQUEST USER INPUT
 30  PRINT "(ENTER 0,0,0 TO END PROGRAM)"
 40  PRINT  "MONTH, DAY, YEAR";
 50  INPUT M,D,Y
 59  REM - TEST FOR END OF PROGRAM
 60  IF M<>0  THEN  100
 70  IF D<>0  THEN  100
 80  IF Y<>0  THEN  100
 90  GOTO  360
 99  REM - NEED TO ADJUST INPUT FOR CALCULATIONS?
100  IF M>2  THEN  130
109  REM - ADJUST INPUT
110  M=M+12
120  Y=Y-1
129  REM - CALCULATE DAY NUMBER
130  N=D+2*M+INT(.6*(M+1))+Y+INT(Y/4)-INT(Y/100)+INT(Y/400)+2
140  N=INT((N/7-INT(N/7))*7+.5)
149  REM - FIND CORRECT DAY NUMBER, TRANSLATE TO DAY, PRINT
150  IF N>0  THEN  180
160  PRINT "SATURDAY"
170  GOTO  340
```

```
180  IF N>1  THEN  210
190  PRINT "SUNDAY"
200  GOTO  340
210  IF N>2  THEN  240
220  PRINT "MONDAY"
230  GOTO  340
240  IF N>3  THEN  270
250  PRINT "TUESDAY"
260  GOTO  340
270  IF N>4  THEN  300
280  PRINT "WEDNESDAY"
290  GOTO  340
300  IF N>5  THEN  330
310  PRINT "THURSDAY"
320  GOTO  340
330  PRINT "FRIDAY"
340  PRINT
349  REM - RESTART PROGRAM
350  GOTO   40
360  END
```

Days between Two Dates

This program calculates the number of days between two given dates. Leap years are taken into account. The program assumes there is one day between today and tomorrow. For instance, there are two days between March 1 and March 3 of the same year.

There are a few precautions to assure the proper use of this program. First, you must be certain to enter the earlier date first. Second, dates must be entered in number form (3, not MARCH) and in the correct order (month, day, year, i.e., 3,17,1976). Commas, not slashes or dashes, must separate the figures. Third, the year must not be abbreviated (1976, not 76), even if both dates are in the same century. Finally, the month entered must not be greater than 12 and the days no greater than the number of days in the particular month. If such is the case, the message UNREAL DATE is printed to alert you to the fact that an unreal date (such as 14,32,1975) has been entered. An incorrect answer is likely to result.

Example:

John's birthdate is August 8, 1951. How many days old will he be on his 30th birthday?

```
:RUN
DAYS BETWEEN TWO DATES

FIRST DATE? 8,8,1951
SECOND DATE? 8,8,1981
DIFFERENCE = 10950 DAYS

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "DAYS BETWEEN TWO DATES"
 20  PRINT
 29  REM - STATEMENTS 30 TO 60 REQUEST USER INPUT
 30  PRINT "FIRST DATE";
 40  INPUT M1,D1,Y1
 50  PRINT "SECOND DATE";
 60  INPUT M2,D2,Y2
 69  REM - SET VARIABLES TO BE USED IN SUBROUTINE
 70  M=M1
 80  D=D1
 90  Y=Y1
100  GOSUB 230
109  REM - SAVE COMPUTED NUMBER OF DAYS IN N
110  N=A
119  REM - SET VARIABLES TO BE USED IN SUBROUTINE
120  M=M2
130  D=D2
140  Y=Y2
```

```
150  GOSUB  230
159  REM - CALCULATE DIFFERENCE AND PRINT
160  N=A-N
170  PRINT "DIFFERENCE =";N;"DAYS"
180  PRINT
189  REM - RESTART OR END PROGRAM?
190  PRINT "MORE DATA (1=YES, 0=NO)";
200  INPUT X
210  IF X=1  THEN  20
219  REM - END PROGRAM
220  GOTO  460
227  REM - SUBROUTINE TO COMPUTE NUMBER OF DAYS SINCE 0,0,0 TO M,D,Y
228  REM - START WITH TEST FOR UNREAL DATE
229  REM - GO TO CORRECT TEST DEPENDING ON NUMBER OF DAYS IN MONTH
230  ON M GOTO  260 , 280 , 260 , 340 , 260 , 340 , 260 , 260 , 340 ,
     260 , 340 , 260
239  REM - IF THIS MESSAGE IS PRINTED THE ANSWER IS PROBABLY INCORRECT
240  PRINT "UNREAL DATE"
249  REM - STOP CALCULATIONS, RETURN TO MAIN PROGRAM
250  RETURN
259  REM - MONTH HAS 31 DAYS
260  IF D>31  THEN  240
270  GOTO  350
279  REM - MONTH IS FEBRUARY; A LEAP YEAR?
280  IF Y/4<>INT(Y/4)  THEN  310
290  IF Y/400=INT(Y/400)  THEN  320
300  IF Y/100<>INT(Y/100)  THEN  320
309  REM - NOT A LEAP YEAR; MONTH HAS 28 DAYS
310  IF D>28  THEN  240
319  REM - A LEAP YEAR; MONTH HAS 29 DAYS
320  IF D>29  THEN  240
330  GOTO  350
339  REM - MONTH HAS 30 DAYS
340  IF D>30  THEN  240
349  REM - TABLE OF NUMBER OF DAYS FROM 1ST OF YEAR TO 1ST OF EACH MON
   TH
350  DATA 0,31,59,90,120,151,181,212,243,273,304,334
360  RESTORE
365  FOR H=1  TO M
369  REM - GET NUMBER OF DAYS FROM JAN 1 TO 1ST OF MONTH FROM DATA TAB
   LE
370  READ A
375  NEXT H
379  REM - COMPUTE NUMBER OF DAYS FROM 0,0,0 TO M,D,Y
380  A=A+Y*365+INT(Y/4)+D+1-INT(Y/100)+INT(Y/400)
389  REM - POSSIBLY A LEAP YEAR?
390  IF INT(Y/4)<>Y/4  THEN  450
409  REM - CONTINUE TEST FOR LEAP YEAR
410  IF Y/400=INT(Y/400)  THEN  430
420  IF Y/100=INT(Y/100)  THEN  450
428  REM - YEAR IS A LEAP YEAR;
429  REM - IF MONTH IS JAN OR FEB ADJUST CALCULATED NUMBER OF DAYS
430  IF M>2  THEN  450
440  A=A-1
449  REM - END OF SUBROUTINE, RETURN TO MAIN PROGRAM
450  RETURN
460  END
```

OPTION

To shorten this program you may wish to omit the test for unreal dates. It should be noted that if a month of more than 12 is entered when this test is omitted, an input error will result. The program lines which may be deleted are listed following the example below.

Example:

How many days are there between July 4 and Christmas?

```
:RUN
DAYS BETWEEN TWO DATES

FIRST DATE? 7,4,1977
SECOND DATE? 12,25,1977
DIFFERENCE = 174 DAYS

MORE DATA (1=YES, 0=NO)? 0

END PROGRAM
```

PROGRAM LISTING

```
  10  PRINT "DAYS BETWEEN TWO DATES"
   :
  90  Y=Y1
 100  GOSUB  350
 109  REM - SAVE COMPUTED NUMBER OF DAYS IN N
   :
 140  Y=Y2
 150  GOSUB  350
 159  REM - CALCULATE DIFFERENCE AND PRINT
   :
 227  REM - SUBROUTINE TO COMPUTE NUMBER OF DAYS SINCE 0,0,0 TO M,D,Y
(Delete lines 228 - 340)
 349  REM - TABLE OF NUMBER OF DAYS FROM 1ST OF YEAR TO 1ST OF EACH MON
      TH
   :
 460  END
```

Anglo to Metric

This program converts a measure given in anglo units to metric units. The conversions available in this program are as follows:

1 Inches to centimeters
2 Feet to centimeters
3 Feet to meters
4 Yards to meters
5 Miles to kilometers
6 Teaspoons to cubic centimeters
7 Tablespoons to cubic centimeters
8 Cups to liters
9 Pints to liters
10 Quarts to liters
11 Gallons to liters
12 Bushels to liters
13 Pecks to liters
14 Ounces to grams
15 Pounds to kilograms
16 Tons to kilograms
17 Degrees Fahrenheit to degrees Celsius

You must provide the value of the anglo measurement and the number of the conversion (1 - 17 as listed above) which you wish to perform.

Example:

Perform the following conversions:

8.5 miles to kilometers
75° Fahrenheit to degrees Celsius
10 gallons to liters

```
:RUN
ANGLO TO METRIC

(TO END PROGRAM ENTER 0)
WHICH CONVERSION DO YOU NEED? 5
VALUE TO BE CONVERTED? 8.5
 8.5 MILES = 13.6765 KILOMETERS

WHICH CONVERSION DO YOU NEED? 17
VALUE TO BE CONVERTED? 75
 75 DEGREES FAHRENHEIT = 23.88888888889 CELSIUS

WHICH CONVERSION DO YOU NEED? 11
VALUE TO BE CONVERTED? 10
 10 GALLONS = 37.85 LITERS

WHICH CONVERSION DO YOU NEED? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "ANGLO TO METRIC"
 20  PRINT
 29  REM - ESTABLISH VARIABLES FOR 17 CONVERSION FACTORS
 30  DIM C(17)
 39  REM - LOOP TO ASSIGN CONVERSION FACTORS INTO C( )
 40  FOR N=1  TO 17
 50  READ C(N)
 60  NEXT N
 69  REM - DATA TABLE OF SEVENTEEN CONVERSION FACTORS
 70  DATA 2.540,30.480,.3048,.9144,1.609,4.929,14.788,.2366,.4732
 80  DATA .9463,3.785,35.24,8.809,28.3495,.4536,907.2,.6214
 89  REM - GET NUMBER OF CONVERSION FROM PROGRAM DESCRIPTION
 90  PRINT "(TO END PROGRAM ENTER 0)"
100  PRINT "WHICH CONVERSION DO YOU NEED";
110  INPUT N
119  REM - END PROGRAM?
120  IF N=0  THEN  540
129  REM - CONVERSION AVAILABLE?
130  IF N>17  THEN  100
140  PRINT "VALUE TO BE CONVERTED";
150  INPUT I
159  REM - PERFORM CONVERSION USING PROPER CONVERSION FACTOR
160  R=I*C(N)
169  REM - DIRECT PROGRAM TO PROPER CONVERSION UNITS, PRINT RESULTS
170  ON N GOTO  180 , 200 , 220 , 240 , 260 , 280 , 300 , 320 , 340 ,
     360 , 380 , 400 , 420 , 440 ,  460 , 480 , 500
180  PRINT I;"INCHES =";R;"CENTIMETERS"
190  GOTO  520
200  PRINT I;"FEET =";R;"CENTIMETERS"
210  GOTO  520
220  PRINT I;"FEET =";R;"METERS"
230  GOTO  520
240  PRINT I;"YARDS =";R;"METERS"
250  GOTO  520
260  PRINT I;"MILES =";R;"KILOMETERS"
270  GOTO  520
280  PRINT I;"TSP. =";R;"CUBIC CENTIMETERS"
290  GOTO  520
300  PRINT I;"TBSP. =";R;"CUBIC CENTIMETERS"
310  GOTO  520
320  PRINT I;"CUPS =";R;"LITERS"
330  GOTO  520
340  PRINT I;"PINTS =";R;"LITERS"
350  GOTO  520
360  PRINT I;"QUARTS =";R;"LITERS"
370  GOTO  520
380  PRINT I;"GALLONS =";R;"LITERS"
390  GOTO  520
400  PRINT I;"BUSHELS =";R;"LITERS"
410  GOTO  520
420  PRINT I;"PECKS =";R;"LITERS"
430  GOTO  520
440  PRINT I;"OUNCES =";R;"GRAMS"
450  GOTO  520
460  PRINT I;"POUNDS =";R;"KILOGRAMS"
```

```
470  GOTO  520
480  PRINT I;"TONS =";R;"KILOGRAMS"
490  GOTO  520
499  REM - CONVERT FROM DEGREES FARENHEIT TO CELSIUS
500  R=(I-32)*5/9
510  PRINT I;"DEGREES FAHRENHEIT =";R;"CELSIUS"
520  PRINT
529  REM - RESTART PROGRAM
530  GOTO  100
540  END
```

Alphabetize

This program alphabetizes a list of words or phrases.

Numbers may be part of an alphanumeric phrase. However, they will not be put into numeric order unless they contain the same number of digits. Numbers with fewer digits must be justified to the right by prefixing zeros. Thus, if the numbers you are sorting range into the hundreds, the number 13 would be entered as 013.

To save memory space, the array at statement 70 should be limited to the maximum number of terms you wish alphabetized. The dimension statement should be altered in the following manner:

```
70 DIM A$(N)
```

where *N* = the number of items to be alphabetized.

Example:

Alphabetize the following names:

Robert Wilson
Susan W. James
Kent Smith
Michael Mitchell
Ann T. McGowan
Alexander Lee II
Mary Mitchell
David Bowers
Steven Evans
Carol Jameson
Linda North

```
:70 DIM A$(11)
:RUN
ALPHABETIZE

(TO END PROGRAM ENTER 0)
NUMBER OF ITEMS? 11
ITEM 1 ? WILSON ROBERT
ITEM 2 ? JAMES SUSAN W.
ITEM 3 ? SMITH KENT
ITEM 4 ? MITCHELL MICHAEL
ITEM 5 ? MCGOWAN ANN T.
ITEM 6 ? LEE ALEXANDER II
ITEM 7 ? MITCHELL MARY
ITEM 8 ? BOWERS DAVID
ITEM 9 ? EVANS STEVEN
ITEM 10 ? JAMESON CAROL
ITEM 11 ? NORTH LINDA
BOWERS DAVID
EVANS STEVEN
JAMES SUSAN W.
JAMESON CAROL
LEE ALEXANDER II
MCGOWAN ANN T.
```

```
MITCHELL MARY
MITCHELL MICHAEL
NORTH LINDA
SMITH KENT
WILSON ROBERT

NUMBER OF ITEMS? 0

END PROGRAM
```

PROGRAM LISTING

```
 10  PRINT "ALPHABETIZE"
 20  PRINT
 30  PRINT "(TO END PROGRAM ENTER 0)"
 40  PRINT "NUMBER OF ITEMS";
 50  INPUT N
 60  IF N=0  THEN  330
 69  REM -LIMIT ARRAY TO MAXIMUM NUMBER OF ITEMS TO BE ENTERED IN ONE RU
 70  DIM A$(25)
 80  FOR I=1  TO N
 90  PRINT "ITEM";I;
100  INPUT A$(I)
110  NEXT I
120  M=N
128  REM - THE SORT TECHNIQUE USED COMPARES DATA ITEMS IN DIMINISHING IN
   REMENTS.
129  REM - THE FIRST PASS COMPARES ITEMS N/2 ELEMENTS APART, THE SECOND
   (N/2)/2 ELEMENTS APART, AND SO ON UNTIL THE INCREMENT IS EXHAUSTED.
130  M=INT(M/2)
140  IF M=0  THEN  280
150  K=N-M
160  J=1
170  I=J
180  L=I+M
190  IF A$(I)<=A$(L)  THEN  250
200  T$=A$(I)
210  A$(I)=A$(L)
220  A$(L)=T$
230  I=I-M
240  IF I>=1  THEN  180
250  J=J+1
260  IF J>K  THEN  130
270  GOTO  170
280  FOR I=1  TO N
290  PRINT A$(I)
300  NEXT I
310  PRINT
320  GOTO  40
330  END
```

OPTION

You may wish your list alphabetized in reverse, or from highest to lowest. The program changes necessary are listed following the example below.

Example:

The scores on a math test range from 82 to 117. Put the students in order according to their scores, from highest to lowest.

```
 89 Bowers
102 Evans
111 James
100 Jameson
 99 Lee
117 McGowan
102 Mitchell
 82 Mitchell
 97 North
 91 Smith
108 Wilson
```

```
:70 DIM A$(11)
:RUN
ALPHABETIZE

(TO END PROGRAM ENTER 0)
NUMBER OF ITEMS? 11
ITEM 1 ? 089 BOWERS
ITEM 2 ? 102 EVANS
ITEM 3 ? 111 JAMES
ITEM 4 ? 100 JAMESON
ITEM 5 ? 099 LEE
ITEM 6 ? 117 MCGOWAN
ITEM 7 ? 102 MITCHELL
ITEM 8 ? 082 MITCHELL
ITEM 9 ? 097 NORTH
ITEM 10 ? 091 SMITH
ITEM 11 ? 108 WILSON
117 MCGOWAN
111 JAMES
108 WILSON
102 MITCHELL
102 EVANS
100 JAMESON
099 LEE
097 NORTH
091 SMITH
089 BOWERS
082 MITCHELL

NUMBER OF ITEMS? 0

END PROGRAM


PROGRAM LISTING
    1  REM - OPTION 190
   10  PRINT "ALPHABETIZE"
    .
    .
    .
  180  L=I+M
  190  IF A$(I)>=A$(L)  THEN  250
  200  T$=A$(I)
    .
    .
    .
  330  END
```

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